

ACCURACY ANALYSIS FOR
A LOWER CONFIDENCE LIMIT PROCEDURE
FOR SYSTEM RELIABILITY

Thomas Robert Gatliffe

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THESIS

ACCURACY ANALYSIS FOR
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FOR SYSTEM RELIABILITY

by

Thomas Robert Gatliffe

September 1976

Thesis Advisor:

W. M. Woods

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20. (Continuation of Abstract)

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FOR SYSTEM RELIABILITY

by

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requirements for the degree of

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ABSTRACT

This thesis examines a proposed empirical method for determining the the $100(1-\alpha)\%$ lower confidence limit for the reliability of a system composed of a mixture of series and parallel connected components for which only component level test data is available. The method is an extension of the Log-gamma procedure originally proposed for series connected systems only. The accuracy of the method is assessed for some representative system reliability constructs using a computer simulation procedure. The simulation results are examined with a view toward identification of accuracy indication parameters which may be estimated prior to the component tests.

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I. INTRODUCTION

Often, in the design and production of complex systems, an important question to be answered early in the process concerns the demonstrable system reliability based upon observed mission test results. Usually only components and minor subassemblies can be tested in any but the very latest stages of production. For reasons of prototype cost, the lack of complete systems for early tests, the expense involved in total system operational tests, and the need to make production decisions as early as feasible, reliance must be shifted to the component test results to provide at least an approximate answer to this question. Component data must be used in spite of the moderately strong assumptions needed to transform such test results into an estimate of overall system reliability.

Since estimates are, by their nature, inexact, it would be reasonable to expect that the most credible estimate would be one which was both conservative and carried with it an indicator of the degree of conservatism associated with its computation. This, of course, is a description fitting a lower confidence limit when referring to estimates of system reliability. Thus, when one speaks of a "100 (1- α)% lower confidence limit" and obtains a value for it, the assertion is that the obtained lower confidence limit value is less than the actual system reliability with probability 1- α .

In 1968, J. R. Borsting and W. M. Woods developed what is known as the Log-Gamma method for computing lower confidence limits for system reliability based upon

component mission test results only [Ref. 1]. As originally developed, the Log-Gamma method was restricted to systems composed of all series connected components. It did, however, offer the advantage of allowing for unequal sample sizes, so the number of component tests could vary arbitrarily from component to component without degrading the procedure accuracy. In addition, no assumption is needed concerning the failure distribution of any component.

Woods has recently proposed a modification to the procedure which, it was hoped, would extend its application to a mixed system of series and parallel connected components. The purpose of this study is to assess the accuracy of the revised procedure when applied to some typical component failure data from several representative series and series-parallel systems. An attempt is also made to discern parameters (constructed from or inherent to the sample data, component reliability goals, testing plan, and system design) which could provide useful indicators of the expected accuracy of the procedure.

II. THE PROCEDURAL MODEL

It should be noted that two rather strong assumptions need to be made with respect to system and component reliabilities. First, it must be assumed that component tests were conducted in a test environment not unlike that encountered in the system operational mission environment profile. Second, no component reliability degradation will result from interactions with other components when assembled into the system. The degree to which these two assumptions are met will have an unknown but potentially great effect upon the validity of the model and indeed upon the whole concept of system reliability prediction from component attributes data.

A. DEVELOPMENT FOR SERIES SYSTEMS

This section gives a modified and elaborated development of the basic reliability model and estimation procedure based upon that presented by Woods and Borsting [Ref. 1].

For a system of k components in logical series, the system reliability, R_s , can be expressed in terms of the individual component reliabilities as the product

$$R_s = \prod_{i=1}^k P_i \quad (2.1)$$

where P_i is the true reliability of the i -th component. From P_i the component unreliability, Q_i , is defined as

$$Q_i = 1 - P_i.$$

Taking the natural logarithm in equation (2.1) and defining the new variable, S:

$$S = -\ln R_s = -\sum_{i=1}^k \ln(1 - Q_i) \quad (2.2)$$

The natural logarithm can be expanded by the infinite series

$$\ln(1 - X) = -\sum_{j=1}^{\infty} (X)^j / j \quad \text{for } 0 \leq X \leq 1$$

$$S = \sum_{i=1}^k \sum_{j=1}^{\infty} (Q_i)^j / j \quad (2.3)$$

$$S = \sum_{i=1}^k [Q_i + Q_i^2/2 + Q_i^3/3 + \dots]$$

For Q_i small, say 0.1 or less, the above series may be approximated by the first two terms with a maximum error of 0.34% of the true value. Thus,

$$S \doteq \sum_{i=1}^k [Q_i + Q_i^2/2] = \sum_{i=1}^k T_i \quad (2.4)$$

where $T_i = Q_i + Q_i^2/2$. For the remainder of the derivation this approximation is treated as an equality.

Appendix A derives an unbiased estimator, \hat{T}_i , for T_i :

$$\hat{T}_i = A_i \hat{Q}_i + B_i [\hat{Q}_i^2] / 2 \quad (2.5)$$

where $A_i = [2N_i - 3] / [2N_i - 2]$

$$B_i = N_i / [N_i - 1]$$

$$\hat{Q}_i = F_i / N_i$$

and, F_i is the number of failures resulting from N_i mission tests conducted on the i -th component.

An unbiased estimator of S will be obtained from

$$\hat{S} = \sum_{i=1}^k \hat{T}_i \quad (2.6)$$

\hat{S} will be an unbiased estimator of S , since it is the sum of the unbiased estimators, \hat{T}_i , of T_i .

Appendix B shows that, for the reasonably expected range of values for Q_i and N_i , the variance of \hat{S} may be approximated

$$\text{Var}[\hat{S}] = \sum_{i=1}^k \text{Var}[\hat{T}_i] \doteq \sum_{i=1}^k [T_i/N_i] \quad (2.7)$$

By intuitively assuming the probability distribution of \hat{S} is of the gamma type with parameters r and θ , an expression can be derived for a confidence interval for R_s .

The gamma probability density may be written as

$$f_{\hat{S}}(x, r, \theta) = \begin{cases} \frac{1}{\Gamma(r) \cdot \theta^r} x^{r-1} e^{-x/\theta} & x \geq 0 \\ 0 & x < 0 \end{cases} \quad (2.8)$$

with mean = $r\theta$ and variance = $r\theta^2$.

To determine the values of r and θ , the following relationships are established:

$$E[\hat{S}] = r\theta \quad \text{from Eq. (2.8)}$$

$$E[\hat{S}] = \sum_{i=1}^k T_i \quad \text{from Eq. (2.4)}$$

$$\text{Var}[\hat{S}] = r\theta^2 \quad \text{from Eq. (2.8)}$$

$$\text{Var}[\hat{S}] = \sum_{i=1}^k \frac{T_i}{N_i}$$

These equations may be solved simultaneously to yield the

following expressions for r and θ :

$$r = \frac{\left[\frac{k}{\sum_{i=1}^k T_i} \right]^2}{\frac{k}{\sum_{i=1}^k \frac{T_i}{N_i}}} \quad \text{and} \quad \theta = \frac{\frac{k}{\sum_{i=1}^k \frac{T_i}{N_i}}}{\frac{k}{\sum_{i=1}^k T_i}}$$

and an estimator, \hat{r} , for r would be

$$\hat{r} = \frac{\left[\frac{k}{\sum_{i=1}^k \hat{T}_i} \right]^2}{\frac{k}{\sum_{i=1}^k \frac{\hat{T}_i}{N_i}}} \quad (2.9)$$

Note that for \hat{S} distributed as $\text{gamma}(r, \theta)$, $2\hat{S}/\theta$ will be distributed as chi-square with $2r$ degrees of freedom [Ref. 2, p. 181]. The following probability statements can then be made:

$$1-\alpha = P \left[\frac{2\hat{S}}{\theta} \geq \chi_{2r, 1-\alpha}^2 \right] \quad (2.10)$$

$$1-\alpha = P \left[\theta \leq \frac{2\hat{S}}{\chi_{2r, 1-\alpha}^2} \right]$$

Since $\theta = E[\hat{S}]/r$ from Eq. (2.8) and $E[\hat{S}] = -\ln R_S$ from Eq. (2.2),

$$1-\alpha = P \left[\frac{-\ln R_S}{r} \leq \frac{2\hat{S}}{\chi_{2r, 1-\alpha}^2} \right]$$

$$1-\alpha = P \left[\ln R_S \leq \frac{-2r\hat{S}}{\chi_{2r, 1-\alpha}^2} \right]$$

Thus the $100(1-\alpha)\%$ lower confidence limit, $R_S^{*(\alpha)}$, for R_S is

$$R_S^{*(\alpha)} = \exp \left[\frac{-2r\hat{S}}{\chi_{2r, 1-\alpha}^2} \right] \quad (2.11)$$

A $100(1-\alpha)\%$ lower confidence limit estimate, $\hat{R}_s^*(\alpha)$, for R_s would be

$$\hat{R}_s^*(\alpha) = \exp\left[\frac{-2\hat{r}\hat{S}}{\chi_{2\hat{r}, 1-\alpha}^2}\right] \quad (2.12)$$

The primary difficulty this formulation presents lies in the non-integer nature of values for $2\hat{r}$ as defined. Although some computer routines are available to approximate chi-square values with non-integer degrees of freedom, most users would find such a requirement difficult to fulfill at least. Therefore $2\hat{r}$ is replaced with the expression

$$[2\hat{r}] = \text{smallest integer} \geq 2\hat{r}$$

This is justified by noting that the ratio of the chi-square value divided by its number of degrees of freedom varies slowly with changes in the degrees of freedom for values of degrees of freedom greater than about five. Further, since $2\hat{r}$ may be changed moderately, by up to 1.0, without great change in the ratio value, it is reasonable to expect that

the probability distribution of $[2\hat{r}]\chi_{[2\hat{r}], 1-\alpha}^2$ has small

variance. This expression can be substituted into equation (2.12) to yield

$$R_s^*(\alpha) = \exp\left[\frac{-[2\hat{r}]\hat{S}}{\chi_{[2\hat{r}], 1-\alpha}^2}\right] \quad (2.13)$$

Since the computer routines available for this research included the capability of computation of chi-square values with non-integer degrees of freedom, some simulation runs were performed using both equations (2.12) and (2.13) for comparison. The results indicated that the effects were limited to the third decimal place at worst and thus had little significance with respect to the procedural accuracy.

B. REVISION FOR SERIES-PARALLEL SYSTEM

Proceeding from the strictly series system procedure developed above, the principal revision for the mixed series-parallel system procedure was to redefine \hat{r}_i and \hat{r} as follows:

$$\hat{T}_i = \begin{cases} A_i \frac{F_i}{N_i} + B_i \frac{F_i^2}{2N_i^2} & i=1,2,\dots,K_1 \\ (1-\hat{R}_i) + (1-\hat{R}_i)^2/2 & i=K_1+1,\dots,K_2 \end{cases} \quad (2.14)$$

$$\hat{r} = \max \left[1.0, \frac{\left[\sum_{i=1}^{K_1+K_2} \hat{T}_i \right]^2}{\sum_{i=1}^{K_1+K_2} \frac{\hat{T}_i}{N_i}} \right] \quad (2.15)$$

$$\hat{S} = \sum_{i=1}^{K_1+K_2} \hat{T}_i \quad (2.16)$$

where A_i , B_i , F_i , and N_i are defined as before

K_1 is the number of series components

K_2 is the number of series-parallel subassemblies interconnected in series

\hat{R}_i is the reliability point estimate for the i -th subassembly determined in the usual manner from the individual point estimates for the subassembly components as derived from the raw failure data.

Although some comparison runs were made using A_i and B_i coefficients for the subassembly contribution to \hat{S} , no significant improvement in accuracy was noted and the use of these coefficients in this manner was abandoned due to the difficulty of computation with varying sample sizes within a subassembly.

C. SPECIAL CIRCUMSTANCES

In the event of no failures in any components or a lack of sufficient failures in subassembly components such that the basic procedure yields an \hat{S} value of zero and, thus, a system reliability lower confidence limit estimate of one, an alternate approach must be employed.

Statistically, the optimum solution might be to continue testing until sufficient component failures have been observed such that the above conditions no longer exist. However, such a solution might also not be economically feasible. Therefore, the simulation included provision for an alternate procedure when \hat{S} equals zero at the end of the programmed number of tests on all components. In addition, several cases were included where such conditions would be expected to exist in a significant number of the simulation runs, in order to observe the efficacy of the alternate procedure.

The derivation of this alternate procedure is given in Appendix C. The procedure may be summarized as follows:

(i) Find the series component or subassembly with the smallest actual or equivalent number of trials, N or N' , respectively. (See section IV.B.1)

(ii) Change the number of component or subassembly failures to F' by the following scheme:

α	F'	
0.2	0.37	NOTE: For best results, N or
0.1	0.25	N' should be at least 10.
0.05	0.16	

If a subassembly was selected, treat the subassembly as a component with F' failures out of N' mission trials for the remainder of this procedure.

(iii) Recompute the lower confidence limit estimate using the basic log-gamma procedure with the revised failure data. This estimate is approximately equivalent to α raised to the $1/N$ power as explained in Appendix C.

III. THE SIMULATION

A. PROCEDURE

The computer simulation program is listed in Appendix F. The following procedure was incorporated into the program:

Step One: Initialize and read in values for the total number of components in series, K_1 , the number of subassemblies in series, K_2 , the number of components within each subassembly, the number of tests performed upon each component, N_i , the true reliability of each component, R_i , and the seed value for the random number generator.

Step Two: For each component in turn, generate the observed number of failures as a binomially distributed random variable, F_i , with parameters N_i and $(1-R_i)$.

Step Three: Compute \hat{T}_i for each series component and subassembly, \hat{S} from the sum of the \hat{T}_i , and \hat{r} according to equations (2.14), (2.15), and (2.16) respectively.

Step four: Compute the resulting estimate of the $100(1-\alpha)\%$ lower confidence limit, $\hat{R}^{(\alpha)}$, according to equation (2.13) for representative values of alpha and store each estimate in a vector array of estimates. Alpha values used in this study were 0.20, 0.10, and 0.05.

Step Five: Repeat Steps Two through Four until the three vector arrays of estimates corresponding to the three

different alpha values are 1000 values in length.

Step Six: Order the elements of each vector array from low to high and determine the sample mean and standard deviation.

Step Seven: Compute the true system reliability, R_s , from the input component reliabilities.

Step Eight: Extract the $100(1-\alpha)$ percentile value from the corresponding array and compare with the true system reliability. If the estimation procedure is accurate these two values should agree. If agreement does not occur, the differences should be small, preferably conservative, and, hopefully, can be made to converge toward zero.

Step Nine: Output all data which was input in Step One in order to insure against input error. Output starting and ending random number generator seed values. Output true system reliability, $100(1-\alpha)$ percentile values, differences from true reliability, sample means and standard deviations, and various percentile values of interest. Provision is made in the program for output of all of the sample values both before and after ordering, if the user desires.

Step Four was performed in two different ways for four of the cases where there was a high frequency of runs without failures, called 'null runs' in the output. The normal way of handling this problem was to discard that run data and repeat it from step one. Thus the complete arrays contained estimates from only those runs which had sufficient failures for the procedure to be valid without the special circumstances modification discussed in Section II.C. In order to observe the effect of utilizing the modification, however, cases 18 through 21 were repeated as cases 30 through 33 with the alternate procedure invoked

whenever \hat{S} equalled zero at the end of step three.

Three proprietary routines were utilized in the program and are therefore not reproduced in the program listing. These routines were used to obtain the binomially distributed random variables, the chi-square function values, and to sort the estimate arrays [Ref. 4].

B. DESCRIPTION OF THE CASES

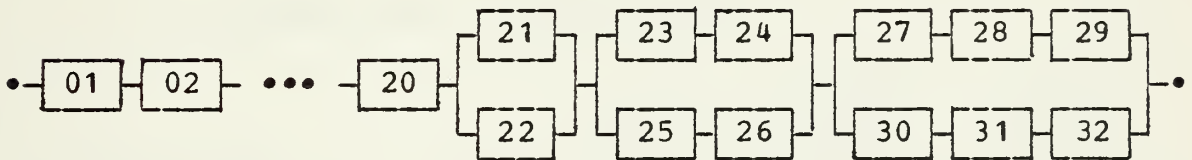
1. General

The cases chosen for simulation represent three different configurations; a series of components connected in series with a series of simple series-parallel subassemblies of components, a strictly series-connected system of components, and a system which incorporates two somewhat unusual but representative "real-world" subassembly reliability relationships. The first configuration was chosen to demonstrate the general performance of the revised estimation procedure with a moderately complex, series-parallel system. The second configuration consisted primarily of some cases which had previously been investigated for the original Log-gamma procedure [Ref. 1]. Since the revised procedure varied somewhat from the originally developed version in the treatment of the wholly series system, a comparison was desirable in order to ascertain whether significant effects had occurred in the procedure accuracy. The third configuration was chosen to observe the robustness of the procedure when applied to a much more complex system which is not easily reducible to a simpler series-parallel reliability representation.

In general, the simulated number of tests was the same for each component within each case. However, this number was varied among cases with the same true system reliability in order to identify its effect upon the procedure accuracy. The two exceptions were case 13 wherein the number of tests varied from component to component and case 14 for which only one value of simulated component tests was used without any corresponding cases with different numbers of tests.

2. Cases One Through Twelve

a. Reliability Block Diagram



b. Test Parameters

(1) Number of Component Tests, N_i .

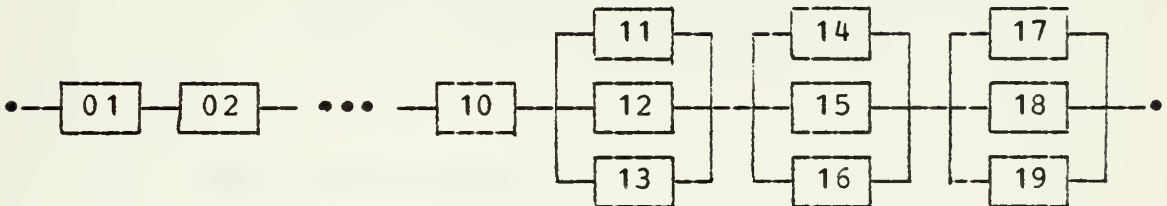
<u>Cases</u>	<u>N_i</u>	
01, 04, 07, 10	20	$i=1,2,\dots,32$
02, 05, 08, 11	50	$i=1,2,\dots,32$
03, 06, 09, 12	100	$i=1,2,\dots,32$

(2) True System and Component Reliabilities, R_s and R_i .

<u>Cases</u>	<u>R_s</u>	<u>R_i</u>
01, 02, 03	.87559	$\left\{ \begin{array}{l} .995 \quad i=1, 2, \dots, 20 \\ .950 \quad i=21, \dots, 32 \end{array} \right.$
04, 05, 06	.79984	$\left\{ \begin{array}{l} .995 \quad i=1, 2, \dots, 20 \\ .900 \quad i=21, \dots, 32 \end{array} \right.$
07, 08, 09	.79167	$\left\{ \begin{array}{l} .990 \quad i=1, 2, \dots, 20 \\ .950 \quad i=21, \dots, 32 \end{array} \right.$
10, 11, 12	.72317	$\left\{ \begin{array}{l} .990 \quad i=1, 2, \dots, 20 \\ .900 \quad i=21, \dots, 32 \end{array} \right.$

3. Case Thirteen

a. Reliability Block Diagram



b. Test Parameters

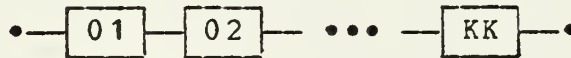
<u>i</u>	<u>N_i</u>	<u>R_i</u>	<u>i</u>	<u>N_i</u>	<u>R_i</u>	<u>i</u>	<u>N_i</u>	<u>R_i</u>
1	150	.995	7	28	.967	13	59	.682
2	90	.985	8	125	.995	14	5	.729
3	75	.979	9	63	.970	15	5	.729
4	100	.988	10	125	.995	16	5	.729
5	125	.982	11	59	.682	17	19	.536
6	18	.980	12	59	.682	18	19	.536
Rs: 0.72329						19	19	.536

c. Discussion

This case corresponds to Case No. 1 of reference 1. The only change was to replace the last three components each with three components in parallel with a subsystem reliability equivalent to the reliability of the replaced component.

4. Cases Fourteen Through Twenty-one and Thirty Through Thirty-three

a. Reliability Block Diagram



b. Test Parameters

(1) Number of Component Tests, N_i .

<u>Cases</u>	<u>N_i</u>	
14	20	$i=1,2,\dots,15$
15, 18, 30	25	$i=1,2,\dots,25$
16, 17, 31	50	$i=1,2,\dots,25$
17, 20, 32	100	$i=1,2,\dots,25$
21, 33	200	$i=1,2,\dots,25$

(2) True System and Component Reliabilities, R_s and R_i .

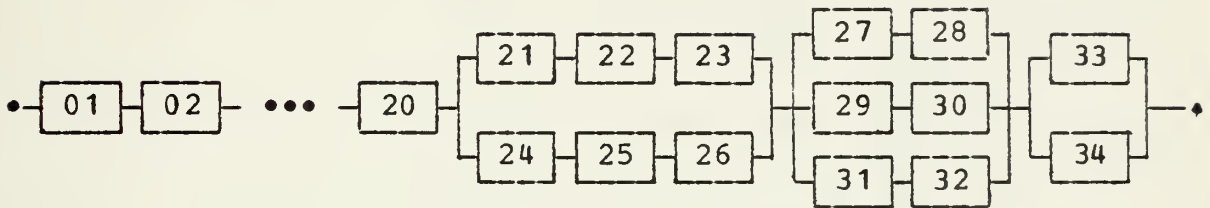
<u>Cases</u>	<u>R_s</u>	<u>R_i</u>
14	.79239	[.995 $i=1,2,\dots,14$.850 $i=15$
15, 16, 17	.88222	.995 $i=1,2,\dots,25$
18, 19, 20, 21 30, 31, 32, 33]	.97530	.999 $i=1,2,\dots,25$

c. Discussion

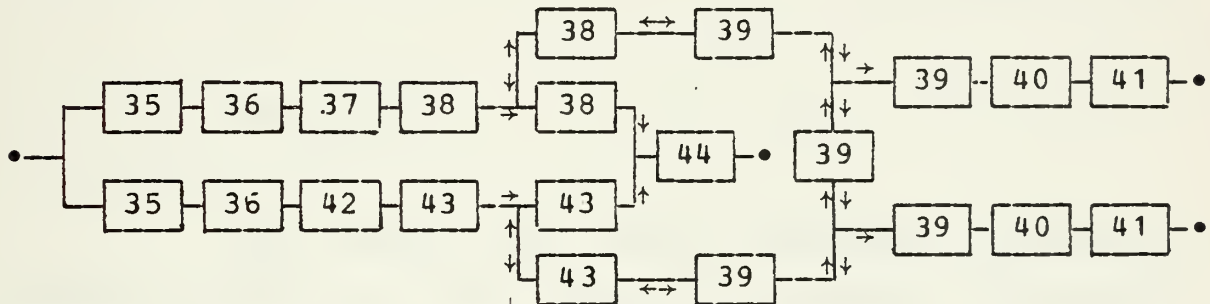
These cases correspond to several cases from Ref. 1 with the exception of cases 21 and 33 of the present study. These were added when it was noted that a large number of runs with zero failures were being observed with as many as 100 tests on each component. Cases 30 through 33 are identical with cases 18 through 21. They are identified as separate cases, since a slightly different procedure, as described in section III.A, was followed in their simulation.

5. Cases Twenty-two Through Twenty-nine

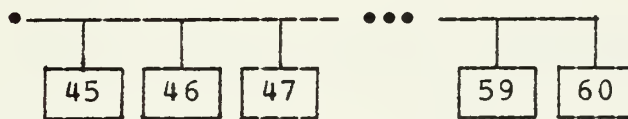
a. Reliability Block Diagram



In logical series with



In logical series with



b. Test Parameters

(1) Number of Component Tests, N_i .

<u>Cases</u>	<u>N_i</u>	
22, 26	20	$i=1, 2, \dots, 60$
23, 27	25	$i=1, 2, \dots, 60$
24, 28	50	$i=1, 2, \dots, 60$
25, 29	100	$i=1, 2, \dots, 60$

(2) True System and Component Reliabilities, R_s and R_i .

<u>Cases</u>	<u>R_s</u>	<u>R_i</u>
22 through 29	(See below)	.999 $i=1,2,\dots,26$
		.886 $i=27,28,\dots,32$
		.900 $i=33,34$
		.999 $i=35,36,\dots,41$
		.990 $i=42,43,44$
22 to 25	.87306	$(.995)^j$ $\left\{ \begin{array}{l} j=i-44 \\ i=45,46,\dots,60 \end{array} \right.$
26 to 29	.83293	$(.990)^j$ $\left\{ \begin{array}{l} j=i-44 \\ i=45,46,\dots,60 \end{array} \right.$

c. Discussion

The second and third subassembly reliability diagrams above were drawn from the Lockheed Reliability Evaluation Plan for the Trident Missile [Ref. 3].

In the second diagram, component 44 and both components 41 must operate successfully for system success. An additional feature of this design is the capability of obtaining successful functioning of component 44 by following a logical path through component 39 and continuing in a reverse direction to reach component 44 from the opposite side of the diagram, if necessary. This capability is illustrated with double arrows to indicate that the logical success path may be traced in either direction for that portion of the reliability diagram. Note also that identically numbered components are, in fact, separate components but identical in operation. Therefore, component test results for a particular component type are assumed to be equally applicable to all components of that type, regardless of the specific application. The reliability

equation for this portion of the diagram, as adapted from reference 3, is as follows:

$$\begin{aligned}
 R_{SS} = M_4 & \left[T_1 T_2 T_3 L_3 M_3 [1 - (1-M_1)(1-L_2 M_2)] \right. \\
 & + (1-T_1) L_1 L_2 L_3 T_3 M_3 [1 - (1-M_2)(1-T_2 M_1)] \left. \right] \\
 & + (1-M_4) \left[M_1 M_2 M_3 T_2 T_3 L_2 L_3 [1 - (1-T_1)(1-L_1)] \right. \\
 & + (1-M_1) M_2 M_3 T_1 T_2 T_3 L_1 L_2 L_3 \\
 & \left. + (1-M_2) M_1 M_3 T_1 T_2 T_3 L_1 L_2 L_3 \right] \quad (3.1)
 \end{aligned}$$

where

$$\begin{aligned}
 T_1 &= R_{35} R_{36} R_{37} R_{38} & M_1 &= R_{38} \\
 T_2 &= R_{38} R_{39} & M_2 &= R_{43} \\
 T_3 &= R_{39} R_{40} R_{41} & M_3 &= R_{44} \\
 L_1 &= R_{35} R_{36} R_{42} R_{43} & M_4 &= R_{39} \\
 L_2 &= R_{43} R_{39} & L_3 &= T_3
 \end{aligned}$$

The third subassembly diagram illustrates another type of reliability problem in that the subassembly reliability is defined in Ref. 3 as follows:

$$R_{SS} = \frac{\sum_{i=45}^{60} R_i}{16}$$

Although these diagrams represent current systems, the reliability values chosen bear no intentional relation to actual values other than that acquired when choosing them on a criteria of reasonableness. Therefore the values used are not classified.

IV. RESULTS AND CONCLUSIONS

A. SIMULATION RESULTS

The simulation output is listed in detail in Appendix D for each run. Each case was simulated three times with different random number generator seed values to insure against the occurrence of non-representative singularities in the observed data. Each simulation is distinguished by a letter suffix, a, b, or c, to the case number in Appendix D. The starting seed used each time was the ending seed value from the previous run. This insured that each run would be independent [Ref. 5].

The demonstrated accuracy of the estimation procedure can be measured in terms of the difference between the 100(1- α) percentile of the simulation results, \hat{R}^* , and the true system reliability, R_s . This difference, labeled D in the output tables, is defined as follows:

$$D = \hat{R}^* - R_s \quad (4.1)$$

As mentioned in Section III.A, the absolute value of D should be small and reflect a conservative error. This desired conservatism is satisfied so long as the sign of D is negative. Otherwise the lower confidence limit estimation procedure would yield overly-optimistic estimates and could not be a valid vehicle for the description of system reliability. The one instance where this condition was not satisfied occurred in Case 13c. However, the value of D was extremely small, represented less than one-half of

one percent error, and differed from the conservative results of the other runs by less than one percent.

It should also be noted that the distributions of the estimates \hat{T}_i , \hat{S} , and \hat{r} are discrete in nature and the values are determined by the number and arrangement of components and the number of component tests. Estimates based upon few failures will exhibit the most highly discrete sample distribution scheme. The effect of this discreteness will generally be to increase the observed error magnitude unless some form of continuity correction can be developed.

B. ANALYSIS OF RESULTS

1. General Characteristics

The simulation output for Cases One through Twelve was expanded to include the complete ordered arrays of estimated confidence limit values in order to establish the nature of the sample. Some properties were evident from inspection. The samples were unimodal and discrete, as expected. Discreteness was most pronounced in the higher valued estimates. The sample distribution appeared nearly symmetric and this supposition was checked by comparing the sample mean and median values and also by plotting complementary quantiles. The first test showed very close agreement throughout. The second test indicated that only minor left skewness exists in most cases. Skewness seemed more pronounced for higher values of system reliability, as might be expected might be suspected, since the distribution is bounded by an upper estimate value of 1.0.

The shape of the distribution was further described

by examining the ratio of the interquartile range divided by the interdecile range and comparing it with the same ratio from the Normal distribution which is about 0.526. The observed values were generally on the order of 0.50 which is in good agreement considering the discrete nature of the sample distribution. Systems designed with high reliability coupled with insufficient testing, such as Cases 18 and 19, exhibited great variations in the ratio, from 0.2 to 1.0, due to the sparsity of different sample values occurring in the pronouncedly discrete sample distribution.

Another useful measure of shape is the sample standard deviation. It is particularly helpful in quantifying the spread of the data in terms of the units of measure. The preceding ratio, on the other hand, was independent of the units. The sample standard deviation, however, gives no indication of skewness nor the discreteness of the underlying distribution. The standard deviation is most useful when these higher order effects are small.

The ability to predict the standard deviation from the test plan and system design parameters would provide an additional measure of the system reliability. This concept evolves from a consideration of the $100(1-\alpha)$ percent lower confidence limit. Of itself, it tells us only that there is a $(1-\alpha)$ probability that the true system reliability is greater than a particular value. The standard deviation of the estimate of this confidence limit can provide a feel for how much the true system reliability might vary from the estimate. For example, given an eighty percent lower confidence limit of 0.78 and a predicted standard deviation of the estimate of 0.005, we can feel fairly certain that the true system reliability does not exceed 0.82. However, had the standard deviation been 0.01 or 0.02, there would be increasing uncertainty about the range of likely values of

true reliability. Therefore, the sample standard deviation has been included in the regression section later in this chapter in an effort to not only predict procedure accuracy, but also estimate spread.

2. Derivation and Discussion of TT

Previous tests of the original Log-Gamma procedure have indicated that the quantity

$$TT = \sum_{i=1}^k (N_i) (Q_i) \quad (4.2)$$

could be used as an indicator of relative accuracy of the procedure, where N_i is the number of mission tests performed upon the i -th component and Q_i is the design or expected unreliability goal for the i -th component [Ref. 1]. The quantity determined in this manner was the total amount of testing relative to the component unreliabilities, or, approximately, the expected number of test failures summed over all components and tests.

A similar measure was sought for the revised procedure examined in this paper. However, a simple expression, such as equation (4.2), could not be directly applied to the mixed series-parallel system. If the equation were used, it could lead to extremely large values of TT , since the parallel components would tend to dominate the sum. This is reasonable to expect, since the components connected in parallel, taken individually, would tend to have greater unreliability than the series components.

The first revised equation proposed for TT can be used whenever all components within each parallel subsystem are given identical numbers of mission tests.

$$TT = \sum_{i=1}^{K1} (Ni) (Qi) + \sum_{i=K1+1}^{K1+K2} (Ni') (Qi') \quad (4.3)$$

where K1 is the number of series components in the system
 K2 is the number of series-parallel subassemblies
 connected in logical series in the system

Ni, Qi are defined as above.

Ni' is the common number of tests given each component
 in the i-th subassembly.

Qi' is the unreliability of the i-th subassembly
 computed from its component unreliability goals using
 standard series-parallel reliability computation techniques.

Equation (4.3), in effect, reduces each subassembly
 to an equivalent 'pseudo-component' connected in series in
 the system. The quantity TT is now approximately equal to
 the expected number of system-affecting failures summed over
 all components, subassemblies, and tests. However, the
 drawback of this equation is the necessity of testing all
 components within a subassembly an identical number of
 times. It would seem unnecessarily restrictive to conduct
 the same number of tests on a medium reliability component
 and on a high reliability component just to satisfy the
 equation.

Therefore an alternate formulation for the term
 (Ni')(Qi') was sought which would provide a meaningful
 measure, and yet would allow unequal amounts of testing
 within subassemblies. The following ad hoc formulation is
 proposed:

Each wholly series-connected branch within a
 subsystem will first be reduced to an equivalent single
 component by the following formula

$$Nb = \frac{\sum_{i \in B} (Ni) (1-Ri)}{1 - Rb} \quad (4.4)$$

where R_b is the classically determined branch reliability goal $R_b = \prod_{i \in B} R_i$ for series-connected components.

R_i is the reliability goal of the i -th component.

B is the set of all components in the branch.

N_b represents an "equivalent" number of branch tests (need not be integer).

The series-connected branch is replaced in the remaining computations by a single equivalent component with reliability goal $R_i = R_b$ and number of mission tests $N_i = N_b$.

After all wholly series-connected branches have been reduced in this manner, each wholly parallel-connected branch in the revised subassembly is reduced to an equivalent component in the following manner:

Compute the equivalent reliability goal

$$R_b = 1 - \prod_{i \in B} (1 - R_i)$$

in the usual manner.

The number of possible combinations of individual test results for the branch components is $\prod_{i \in B} N_i$, which is set equal to the same number of possible combinations obtained if each component had been tested N_b times. Thus the equivalent number of mission tests, N_b , is computed as the geometric mean of the component tests

$$N_b = \left[\prod_{i \in B} N_i \right]^{1/m}$$

The above procedure is repeated until the entire subassembly is reduced to a single equivalent component, i' ,

with its equivalent number of mission trials, Ni' , and equivalent unreliability goal, $Qi' = 1-Ri'$. TT may then be computed as before:

$$TT = \sum_{i=1}^{K1} (Ni) (Qi) + \sum_{i=K1+1}^{K1+K2} (Ni') (Qi') \quad (4.3)$$

3. Accuracy Prediction

Appendix E contains the equations and methodology used to attempt to relate the accuracy measure, D, and the estimate standard deviation to the quantity TT. The principal conclusion to be drawn from the results is that accuracy is a predictable quantity only with respect to particular system configurations for which models have been generated and tested as in this paper. Accuracy seems an unstable variable from configuration to configuration, although it appears to be fairly well behaved within any one configuration.

With respect to the estimate standard deviation, the value appears to be related more to the number of component tests than to the expected number of system affecting failures. Reduction in the magnitude of the standard deviation by about 0.01 appears to require almost double the number of component tests. However, the magnitudes are generally less than 0.07 for component test sample sizes of 20 or more. The standard deviation values also appear to be fairly insensitive to changes in system configuration.

4. Comparison with Earlier Results.

Cases 13 through 20 were taken from Ref. 1 in order to provide a basis for checking relative accuracy when

extending the Log-gamma procedure beyond simple series systems. Unfortunately, an apparent shortcoming of the original simulation makes the 1968 data suspect.

The original results tended to be too optimistic for those cases, in particular, where the number of component tests was moderately low and the true component reliabilities were high. Significantly high estimates at the $100(1-\alpha)$ percentile were observed in the cases corresponding to Cases 13, 15, 18, 19, and 20 of the present study. Although, on this basis, the comparison is very favorable to the present procedure, it is thought that the original simulation results are in error with respect to the procedure. Reference 1 does not specifically identify the random number generator used, however some uniform random variate generators do not perform well for very small or very large numbers near the boundaries, such as would be encountered in the simulation procedure used in the original study. The present simulation binomial random variate generator was tested prior to use to insure proper performance throughout the range of simulation values used.

Of the remaining cases (14, 16, and 17) the original results were much closer to the true system reliability in the first two cases and comparable in the last case. However, these results cannot be considered meaningful in light of the suspected problems with the random number generator.

In addition, many instances of cases found in Ref. 1 had prima-facia unreasonable results for the $100(1-\alpha)$ percentile values, in that they were considerably higher than the upper bound imposed by the zero-failure, binomial system test treatment discussed in Appendix C.

Another difference between the original study and

the present one is the use of an arbitrary continuity correction in the earlier procedure. This could also have been a contributor to the excess optimism in the reported simulation results.

In light of the foregoing paragraphs, no comparison with the results of this study was deemed worthwhile.

C. CONCLUSIONS

The primary conclusion to be drawn from this study is that the Log-gamma lower confidence level procedure for series-parallel systems can be accurate. How accurate it will be in any specific application is a function of the magnitude of the component testing program, the individual component reliabilities, and the complexity of the system. In addition, of course, the validity of the assumptions concerning component interaction and mission environment will also bear directly upon the accuracy of any procedure which purports to estimate system mission reliability from component test results. The effects of these assumptions, however will be controlled by the proper design of the component test procedures and are not addressed in this study.

For the cases considered the Log-gamma procedure tends to be conservative, which is necessary if one takes the position that pleasant surprises with better than expected performance cost less than unpleasant surprises in the form of degraded demonstrated reliability. Of course, too much underestimation of system reliability could drive up the development costs in an unnecessary search for a much more reliable design.

The accuracy indicator TT developed in Section IV.B.2 can be used as a rough gauge of expected procedure accuracy. The following scheme is suggested for minimum values of TT for fairly good expected accuracy:

Approximate Order of Accuracy	$\alpha = 0.20$	$\alpha = 0.10$	$\alpha = 0.05$
0.02	6.1	12.8	19.2
0.01	11.7	23.6	32.6

However, since procedure accuracy is so demonstrably sensitive to variation in the system configuration, the best approach prior to use of the Log-gamma procedure is to perform a simulation as described in Section III.A for the particular system design to be exercised. Such a simulation could be used to evaluate many different combinations of component test plans and also support the claimed accuracy of the lower confidence limit derived from the adopted test plan results. The investment in such a simulation effort should not be excessive from the standpoint of computer time, since the most complex cases examined in this study required less than five minutes of Central Processor Unit (CPU) time from start to end. Without the support of such a simulation specific to the system, the accuracy of the procedure can be predicted only very approximately.

For extremely reliable systems, for which few failures are expected with large amounts of testing, the binomial lower confidence limit procedure developed in Appendix C might provide the desired magnitude of the lower confidence limit without the extra testing required to achieve the desired accuracy of the Log-gamma procedure. In fact, for cases of one failure or less, the binomial procedure is probably preferable on the basis of both theoretical rigor and ease of computation.

Finally, the matter of a continuity correction should be investigated. An appropriately determined continuity correction procedure could conceivably reduce the component testing requirements for the same expected procedure accuracy.

APPENDIX A

DERIVATION OF UNBIASED ESTIMATE FOR T_i

For

$$T_i = Q_i + Q_i^2/2$$

an unbiased estimator is desired, based upon the component failure data. Let F_i be the number of failures observed in N_i mission tests on the i -th component. The expected value of F_i is $N_i Q_i$ and an unbiased estimator, \hat{Q}_i , for Q_i is F_i/N_i , where F_i is binomially distributed with parameters Q_i and N_i ; $Q_i = 1 - R_i$; and R_i is the true reliability of the i -th component.

An estimator, \hat{T}_i , for T_i will be chosen of the form

$$\hat{T}_i = A_i[\hat{Q}_i] + B_i[\hat{Q}_i^2]/2 \tag{A.1}$$

and the constants A_i and B_i chosen so as to make the estimator unbiased, that is the expected value of \hat{T}_i is equal to T_i ,

$$E[\hat{T}_i] = T_i = Q_i + Q_i^2/2 \tag{A.2}$$

$$\begin{aligned} &= A_i E[\hat{Q}_i] + (B_i/2) E[\hat{Q}_i^2] \\ &= (A_i/N_i) E[F_i] + (B_i/2N_i^2) E[F_i^2] \\ &= (A_i/N_i)[N_i Q_i] \\ &\quad + (B_i/2N_i^2)[N_i Q_i - N_i Q_i^2 + N_i^2 Q_i^2] \\ &= A_i Q_i + B_i Q_i/2N_i - B_i Q_i^2/2N_i + B_i Q_i^2/2 \\ &= [A_i + B_i/2N_i]Q_i + [B_i/2 - B_i/2N_i]Q_i^2 \tag{A.3} \end{aligned}$$

Equating equation (A.3) term by term with equation (A.2) yields the following two equations

$$A_i + B_i/2N_i = 1$$

and $B_i - B_i/N_i = 1$

which reduce to expressions for the unbiased estimator constants:

$$A_i = (2N_i - 3)/(2N_i - 2) \tag{A.4}$$

and $B_i = N_i/(N_i - 1) \tag{A.5}$

APPENDIX B

APPROXIMATION OF $\text{Var}[\hat{T}_i]$ BY T_i/N_i

A. DERIVATION OF $\text{Var}[\hat{T}_i]$

The variance of \hat{T}_i is derived as follows:

From appendix A,

$$\hat{T}_i = A_i \hat{Q}_i + (B_i \hat{Q}_i^2) / 2 \quad (\text{B.1})$$

thus,
$$\hat{T}_i^2 = A_i^2 \hat{Q}_i^2 + A_i B_i \hat{Q}_i^3 + (B_i^2 \hat{Q}_i^4) / 4 \quad (\text{B.2})$$

$$\text{Var}[\hat{T}_i] = E[\hat{T}_i^2] - E^2[\hat{T}_i] \quad (\text{B.3})$$

$$\begin{aligned} &= E[\hat{T}_i^2] - T_i^2 \\ &= (A_i^2 / N_i^2) E[F_i^2] + (A_i B_i / N_i^3) E[F_i^3] \\ &\quad + (B_i^2 / 4 N_i^4) E[F_i^4] \\ &\quad - [Q_i^2 + Q_i^3 + (Q_i^4 / 4)] \end{aligned} \quad (\text{B.4})$$

Using the moment generating functions for the binomially distributed variable F_i , with parameters Q_i and N_i , yields

$$E[F_i] = N_i Q_i \quad (\text{B.5})$$

$$E[F_i^2] = N_i Q_i + N_i Q_i^2 (N_i - 1) \quad (\text{B.6})$$

$$E[F_i^3] = N_i Q_i + N_i Q_i^2 (N_i - 1) [3 + Q_i (N_i - 2)] \quad (\text{B.7})$$

$$\begin{aligned} E[F_i^4] &= N_i Q_i^4 (N_i - 1) (N_i - 2) (N_i - 3) + 7 N_i Q_i^2 (N_i - 1) \\ &\quad + 6 N_i Q_i^3 (N_i - 1) (N_i - 2) + N_i Q_i \end{aligned} \quad (\text{B.8})$$

Substituting the moment equations (B.5) through (B.8) into equation (B.4) and simplifying the resulting expression yields the following equation for the variance of \hat{T}_i :

$$\begin{aligned} \text{Var}[\hat{T}_i] = & \frac{4Ni^2 - 3Ni + 1}{4Ni(Ni-1)^2} Q_i + \frac{3Ni-4}{2Ni(Ni-1)} Q_i^2 \\ & - \frac{1}{Ni-1} Q_i^3 - \frac{2Ni-3}{2Ni(Ni-1)} Q_i^4 \end{aligned} \quad (\text{B.9})$$

or, in terms of A_i , B_i , N_i , and Q_i :

$$\begin{aligned} \text{Var}[\hat{T}_i] = & \frac{Bi^2 + 2Ni^2 Bi (Ai + Bi)}{4Ni^3} Q_i + \frac{2Ai + 1}{2Ni} Q_i^2 \\ & - \frac{Bi}{Ni} Q_i^3 - \frac{Ai}{Ni} Q_i^4 \end{aligned} \quad (\text{B.10})$$

To test the approximation of $\text{Var}[\hat{T}_i]$ by T_i/N_i , equation (B.9) was solved with representative values of Q_i and N_i and compared with $E[\hat{T}_i]/N_i$. The percent error of the approximation was computed for each value and is given in table B-1. As expected, increasing N_i or decreasing Q_i improves the accuracy of the approximation.

By way of comparison, table B-2 shows the relative accuracy of this same approximation when T_i is defined as the first three, vice two, terms of the logarithmic series expansion of S [Eq. (2.3)]. In this case the unbiased estimator for T_i will be

$$\hat{T}_i = A_i' \hat{Q}_i + B_i' \hat{Q}_i^2 / 2 + C_i' \hat{Q}_i^3 / 3 \quad (\text{B.11})$$

where

$$A_i' = \frac{6(Ni-1)(Ni-2) - 3(Ni-4) - 2}{6(Ni-1)(Ni-2)}$$

$$B_i' = \frac{Ni(Ni-4)}{(Ni-1)(Ni-2)}$$

$$C_i' = \frac{Ni^2}{(Ni-1)(Ni-2)}$$

Qi	Ni = 10	Ni = 50	Ni = 100
0.001	-5.08%	-1.10%	-0.60%
0.003	-5.25%	-1.29%	-0.79%
0.006	-5.52%	-1.58%	-1.09%
0.01	-5.87%	-1.96%	-1.47%
0.03	-7.50%	-3.73%	-3.26%
0.06	-9.64%	-6.07%	-5.62%
0.10	-12.0%	-8.65%	-8.23%
0.20	-15.8%	-12.9%	-12.5%
0.30	-17.0%	-14.4%	-14.1%
0.40	-15.8%	-13.4%	-13.1%
0.50	-12.0%	-9.67%	-9.38%
0.60	-4.77%	-2.40%	-2.11%
0.70	+7.70%	+10.3%	+10.6%

Table B-1. Error Percentage of approximation for Var[Ti] Using Two-term Form of Ti.

Qi	Ni = 10	Ni = 50	Ni = 100
0.001	-0.06%	-0.05%	-0.05%
0.003	-0.17%	-0.15%	-0.15%
0.006	-0.33%	-0.31%	-0.30%
0.01	-0.56%	-0.51%	-0.51%
0.03	-1.68%	-1.54%	-1.53%
0.06	-3.36%	-3.09%	-3.06%
0.10	-5.57%	-5.11%	-5.06%

Table B-2. Error Percentage of approximation for Var[Ti] Using Three-term Form of Ti.

Due to time constraints and the general acceptability of the values noted in table B-1, the use of the more complex definition of Ti was left for future study. However, figure B-1 plots the accuracy as a function of Qi, Ni, and form of Ti.

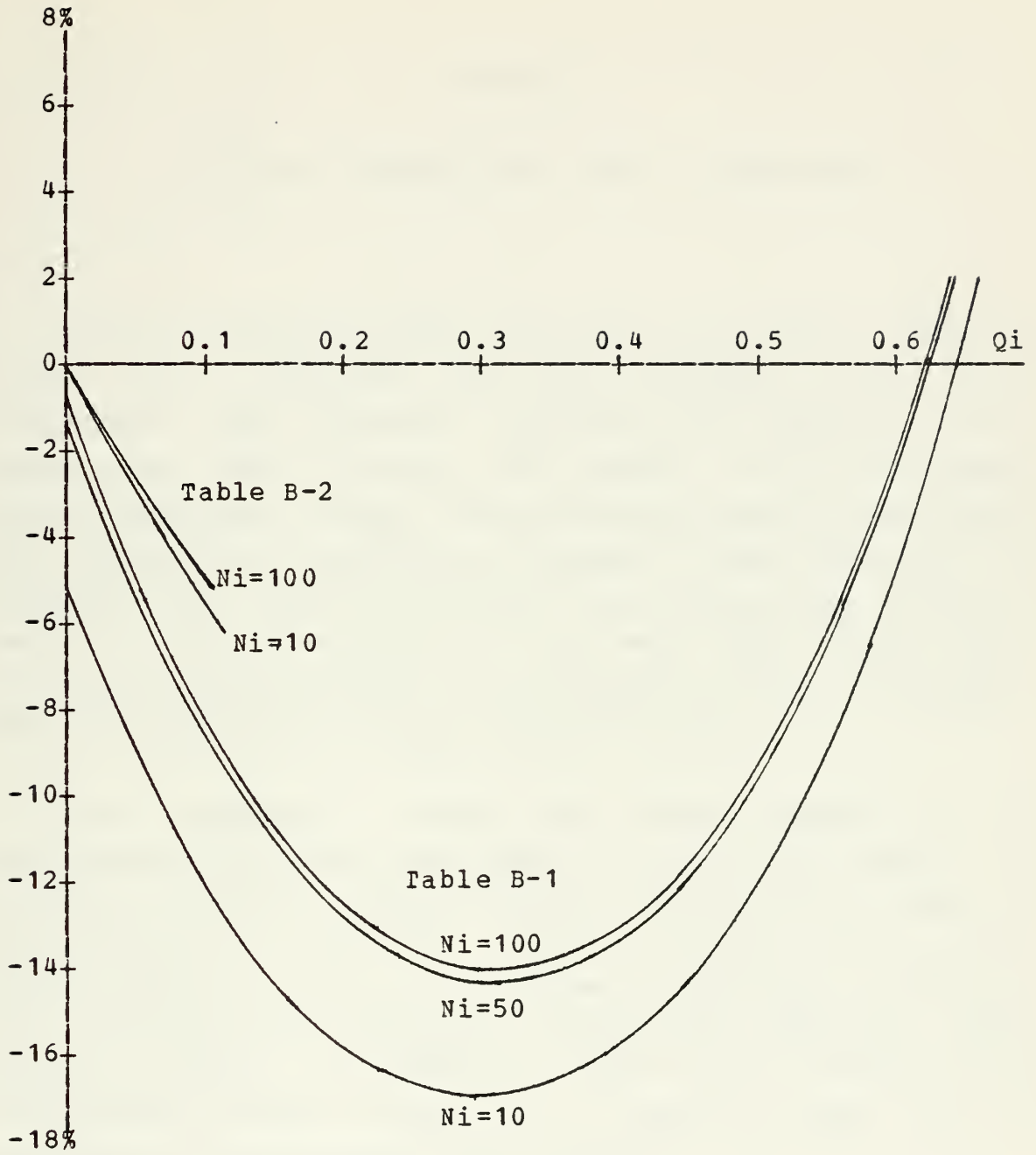


Figure B-1. PERCENTAGE ERROR OF APPROXIMATION FOR $\text{VAR}[\hat{T}_i]$ VS. COMPONENT UNRELIABILITY.

APPENDIX C

SPECIAL CONSIDERATIONS WHEN \hat{S} EQUALS ZERO

When all components have completed the course of mission tests and no failures have been observed along some reliability path through the system, the estimator \hat{S} will equal zero. This, in turn, will result in the log-gamma lower confidence limit estimate being equal to one unless the procedure is modified to include some "virtual" or "pseudo" test failure value for the computations when this occurs. The alternative is to seek some other lower confidence limit procedure which applies with zero failures. Such an alternative is also suggested below.

For simplicity, assume all series components and subassemblies have been subjected to an equal number of mission trials, N , without failure. This may be thought of as equivalent to conducting N mission trials on the system without failure, subject to the usual assumptions concerning test environment and component interaction effects. When unequal numbers of component trials are involved, the conservative course of action is to choose N equal to the smallest of the component trial values.

Now the number of system mission trial successes, W , can be related to system mission reliability, R_s , by the binomial distribution formula

$$\sum_{j=W}^N \binom{N}{j} (R_s)^j (1 - R_s)^{N-j} = \text{Prob}(\# \text{ successes} \geq W) \quad (\text{C.1})$$

where N = number of mission trials

The 100(1-α)% Lower Confidence Limit Estimate for system reliability would then be the solution for R* in the following formula:

$$\sum_{j=W}^N \binom{N}{j} (R^*)^j (1 - R^*)^{N-j} = \alpha \quad (C.2)$$

which, when W = N, reduces to

$$(R^*)^N = \alpha \quad \text{or} \quad R^* = (\alpha)^{1/N}$$

This result may then be substituted into the Log-gamma procedure formula for $\hat{R}^*(\alpha)$ and solved for the number of equivalent component failures which would yield the same estimate of system reliability.

$$\hat{R}^*(\alpha) = \exp \left[\frac{-[2\hat{r}] \hat{S}}{\chi^2_{[2\hat{r}], 1-\alpha}} \right] = (\alpha)^{1/N} \quad (C.3)$$

where

$$\hat{S} = \sum_{i=1}^K \hat{T}_i, \quad \hat{r} = \max \left[1.0, \frac{\left[\sum_{i=1}^K \hat{T}_i \right]^2}{\sum_{i=1}^K \frac{\hat{T}_i}{N_i}} \right]$$

which reduce to $\hat{S} = \hat{T}_i$ and $\hat{r} = \max[1, (N_i)(\hat{T}_i)]$, since only one component or subassembly will make a non-zero contribution to the sums.

Now, if $\hat{r} = 1.0$, then $(N_i)(\hat{T}_i) \leq 1.0$ and $\hat{T}_i \leq 1/N_i$. Also recall that

$$\hat{T}_i = (A_i)(\hat{Q}_i) + (B_i/2)(\hat{Q}_i^2) \quad (2.5)$$

where

$$A_i = \frac{2N_i - 3}{2(N_i - 1)}, \quad B_i = \frac{N_i}{N_i - 1}, \quad \hat{Q}_i = \frac{F_i}{N_i}$$

Or, equivalently,

$$(B_i/2N_i^2) (F_i^2) + (A_i/N_i) (F_i) - \hat{T}_i = 0 \quad (C.4)$$

Substituting for A_i , B_i and the earlier inequality for \hat{T}_i ,

$$\left[\frac{N_i}{2(N_i-1)(N_i^2)} \right] (F_i^2) + \left[\frac{2N_i-3}{2(N_i-1)(N_i)} \right] (F_i) - \frac{1}{N_i} \leq 0$$

which reduces to

$$F_i^2 + (2N_i-3)F_i - 2(N_i-1) \leq 0$$

which, in turn, can be factored as

$$(F_i - 1)[F_i + 2(N_i-1)] \leq 0 \quad (C.5)$$

The only positive values for F_i which can satisfy this last equation are F_i less than or equal to one. This means that for any size N_i , the value of $[2\hat{r}]$ will be two. We can now solve the reliability estimate equation for F_i^* , the number of equivalent failures for the Log-gamma method.

$$\exp \left[\frac{-[2\hat{r}] \hat{S}}{\chi_{[2\hat{r}], 1-\alpha}^2} \right] = (\alpha)^{1/N} \quad (C.6)$$

$$\hat{S} = \left[\frac{\chi_{2, 1-\alpha}^2}{2N} \right] (\ln \alpha) = \frac{A_i (F_i^*)}{N} + \frac{B_i (F_i^*)^2}{2N^2} \quad (C.7)$$

which reduces to

$$F_i^{*2} + (2N-3)F_i^* + (N-1)(\ln \alpha) \left[\chi_{2, 1-\alpha}^2 \right] = 0 \quad (C.8)$$

This quadratic may be solved for F_i^* as a function of N and α . The results are listed in table C-1. From the table, choosing F_i^* at the $N = 10$ level will be conservative for all N greater than 10, yet will yield relatively accurate results for all larger N due to the slow rate of change of F_i^* with respect to N .

N	$\alpha=0.2$	$\alpha=0.1$	$\alpha=.05$
5	0.3888	0.2670	0.1713
10	0.3721	0.2531	0.1610
20	0.3652	0.2475	0.1570
100	0.3603	0.2435	0.1542
1000	0.3593	0.2427	0.1536

Table C-1. Solution for F_i^* with
Various N and α Values.

APPENDIX D

TABULATED SIMULATION RESULTS

The following pages contain two tables listing the principal simulation output data. The tables are presented concurrently for each case, one case per page. The cases are listed in alpha-numeric order. The following list of terms applies:

- Case: The case number as explained in Section III.B. The letter suffix distinguishes between different runs of the same case using different seeds for the binomial random variate generating function.
- KK: The total number of components in the system.
- TT: The accuracy indicator as discussed in section IV.B.
- TM: The modified accuracy indicator described in Appendix E.
- Rs: True system reliability.
- CL: Confidence Level, $CL = 100(1-\alpha)$.
- \hat{R}^*0 : The $100(1-\alpha)$ percentile of the simulation estimates of the $100(1-\alpha)\%$ lower confidence limit for system reliability.
- D: The primary accuracy measure. As discussed in Section IV.A, $D = \hat{R}^*0 - R_s$.
- Mean: The sample mean of the estimates.
- S.D.: The sample standard deviation of the estimates.

Null Runs: The number of runs which occurred during the simulation for which \hat{S} equalled zero. Refer to sections II.C and III.A and Appendix C for further elaboration.

Case	KK	TT	TM	RS	CL	\hat{R}^*	D	Mean	S.D.
01a	32	2.67	1.73	0.8756	80.0	0.8206	-0.0550	0.7728	0.0668
					90.0	0.7452	-0.1304	0.6896	0.0608
					95.0	0.6457	-0.2299	0.5897	0.0586
01b	32	2.67	1.73	0.8756	80.0	0.8193	-0.0563	0.7727	0.0646
					90.0	0.7464	-0.1292	0.6892	0.0584
					95.0	0.6461	-0.2295	0.5894	0.0560
01c	32	2.67	1.73	0.8756	80.0	0.8235	-0.0521	0.7721	0.0706
					90.0	0.7500	-0.1256	0.6893	0.0652
					95.0	0.6551	-0.2205	0.5898	0.0624

Table D-1. Simulation Results.

Case	Rs	Null Runs	CL	Lower Confidence Limit Estimate Percentile Values						
				Minimum	10 %	25 %	Median	75 %	90 %	Maximum
01a	0.87559	3	.80 .90 .95	.53944 .48173 .37693	.69032 .62211 .53409	.73043 .65534 .56606	.77577 .68976 .59030	.81214 .71470 .61304	.84858 .74522 .63225	.98884 .97652 .95232
01b	0.87559	2	.80 .90 .95	.58990 .52912 .37693	.69065 .62342 .53492	.73087 .65790 .56598	.77493 .68654 .58906	.81276 .71696 .61356	.84287 .74642 .63413	.98884 .97652 .95232
01c	0.87559	4	.80 .90 .95	.54338 .48590 .37693	.68722 .61906 .53220	.72971 .65396 .56440	.77453 .68826 .58946	.81283 .71614 .61321	.84552 .75004 .63796	.97779 .95352 .90680

Table D-2. Additional Simulation Output

Case	KK	TT	TM	RS	CL	\hat{R}^*	D	Mean	S. D.
02a	32	6.68	4.33	0.8756	80.0	0.8609	-0.0147	0.8232	0.0442
					90.0	0.8416	-0.0340	0.7883	0.0434
					95.0	0.8127	-0.0629	0.7525	0.0407
02b	32	6.68	4.33	0.8756	80.0	0.8593	-0.0163	0.8220	0.0440
					90.0	0.8411	-0.0345	0.7871	0.0431
					95.0	0.8108	-0.0648	0.7512	0.0401
02c	32	6.68	4.33	0.8756	80.0	0.8581	-0.0175	0.8219	0.0419
					90.0	0.8382	-0.0374	0.7871	0.0413
					95.0	0.8079	-0.0677	0.7515	0.0388

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Confidence Limit			Estimate Percentile Values			
				Minimum	10 %	25 %	Median	75 %	90 %	Maximum
02a	0.87559	0	.80 .90 .95	.65891 .62550 .59525	.76871 .73444 .70164	.79558 .76068 .72700	.82406 .78970 .75476	.85334 .81891 .78227	.87724 .84163 .80256	.93761 .89509 .83969
02b	0.87559	0	.80 .90 .95	.67925 .64552 .61468	.76328 .72870 .69577	.79249 .75819 .72474	.82399 .78935 .75520	.85253 .81748 .78037	.87684 .84113 .80123	.95942 .91599 .83924
02c	0.87559	0	.80 .90 .95	.69563 .66188 .63083	.76882 .73484 .70172	.79494 .76030 .72656	.82324 .78868 .75330	.85206 .81707 .78017	.87362 .83819 .79999	.92467 .88568 .83678

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	RS	CL	\hat{R}^*	D	Mean	S.D.
03a	32	13.4	8.67	0.8756	80.0	0.8691	-0.0065	0.8425	0.0325
					90.0	0.8628	-0.0128	0.8220	0.0330
					95.0	0.8535	-0.0221	0.8025	0.0329
03b	32	13.4	8.67	0.8756	80.0	0.8655	-0.0101	0.8392	0.0323
					90.0	0.8573	-0.0183	0.8187	0.0328
					95.0	0.8476	-0.0280	0.7991	0.0327
03c	32	13.4	8.67	0.8756	80.0	0.8678	-0.0078	0.8421	0.0314
					90.0	0.8603	-0.0153	0.8216	0.0319
					95.0	0.8520	-0.0236	0.8021	0.0319

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Confidence Limit			Estimate Percentile Values			
				Minimum	10 %	25 %	Median	75 %	90 %	Maximum
03a	0.87559	0	.80	.75991	.80069	.82015	.84283	.86422	.88257	.93326
			.90	.73837	.77952	.79940	.82246	.84404	.86278	.91309
			.95	.71882	.75992	.77990	.80288	.82450	.84349	.89037
03b	0.87559	0	.80	.73571	.79738	.81835	.83998	.86190	.87707	.93497
			.90	.71376	.77624	.79741	.81930	.84186	.85726	.91529
			.95	.69399	.75672	.77787	.79968	.82239	.83799	.89310
03c	0.87559	0	.80	.72448	.80323	.82190	.84334	.86270	.87996	.94266
			.90	.70250	.78193	.80095	.82283	.84268	.86025	.92343
			.95	.68279	.76221	.78131	.80313	.82319	.84080	.90110

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	RS	CL	R*	D	Mean	S.D.
04a	32	4.59	2.97	0.7998	80.0	0.7555	-0.0443	0.6962	0.0680
					90.0	0.6988	-0.1010	0.6250	0.0596
					95.0	0.6196	-0.1802	0.5506	0.0488
04b	32	4.59	2.97	0.7998	80.0	0.7524	-0.0474	0.6948	0.0673
					90.0	0.6975	-0.1023	0.6247	0.0605
					95.0	0.6200	-0.1798	0.5525	0.0501
04c	32	4.59	2.97	0.7998	80.0	0.7544	-0.0454	0.6953	0.0664
					90.0	0.6956	-0.1042	0.6247	0.0588
					95.0	0.6161	-0.1837	0.5515	0.0471

Table D-1. Simulation Results (continued).

Case	RS	Null Runs	CL	Lower Minimum	Confidence Limit			Estimate	Percentile Values		
					10 %	25 %	75 %		90 %	Maximum	
04a	0.79984	0	.80 .90 .95	.48972 .43625 .38537	.61041 .54882 .48419	.65058 .58596 .52212	.69590 .62685 .55678	.74771 .67005 .58542	.78197 .69881 .60847	.87023 .77064 .65042	
04b	0.79984	0	.80 .90 .95	.44002 .39024 .34712	.60848 .54672 .48617	.65274 .58755 .52372	.69752 .62925 .55975	.74341 .66842 .58732	.77654 .69749 .60935	.87521 .75399 .65509	
04c	0.79984	0	.80 .90 .95	.47640 .42249 .37508	.60950 .54783 .48818	.65157 .58604 .52285	.69646 .62869 .55888	.74662 .66911 .58553	.77755 .69563 .60642	.88866 .77875 .65455	

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	RS	CL	\hat{R}^*	D	Mean	S.D.
05a	32	11.5	7.43	0.7998	80.0	0.7839	-0.0159	0.7451	0.0450
					90.0	0.7689	-0.0309	0.7108	0.0448
					95.0	0.7458	-0.0541	0.6783	0.0437
05b	32	11.5	7.43	0.7998	80.0	0.7816	-0.0183	0.7449	0.0429
					90.0	0.7631	-0.0367	0.7106	0.0428
					95.0	0.7452	-0.0547	0.6781	0.0416
05c	32	11.5	7.43	0.7998	80.0	0.7832	-0.0167	0.7454	0.0464
					90.0	0.7696	-0.0302	0.7111	0.0461
					95.0	0.7460	-0.0538	0.6786	0.0450

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Minimum	Confidence 10 %	Limit 25 %	Estimate Median	Percentile Values		
								75 %	90 %	Maximum
05a	0.79984	0	.80 .90 .95	.59499 .56231 .53328	.68927 .65536 .62407	.71713 .68309 .65145	.74444 .71020 .67778	.77731 .74243 .70872	.80302 .76893 .73432	.86177 .82783 .79186
05b	0.79984	0	.80 .90 .95	.60950 .57666 .54698	.68998 .65578 .62435	.71483 .68059 .64876	.74415 .71000 .67767	.77564 .74129 .70816	.79795 .76313 .72939	.88186 .84399 .80112
05c	0.79984	0	.80 .90 .95	.54462 .51291 .48516	.68834 .65416 .62324	.71679 .68272 .65105	.74954 .71535 .68300	.77669 .74256 .70906	.80420 .76962 .73594	.85368 .82016 .78529

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	RS	CL	\hat{R}^*	D	Mean	S.D.
06a	32	22.9	14.9	0.7998	80.0	0.7891	-0.0107	0.7625	0.0314
					90.0	0.7810	-0.0188	0.7409	0.0318
					95.0	0.7716	-0.0282	0.7213	0.0317
06b	32	22.9	14.9	0.7998	80.0	0.7886	-0.0113	0.7619	0.0320
					90.0	0.7828	-0.0170	0.7403	0.0322
					95.0	0.7748	-0.0250	0.7207	0.0322
06c	32	22.9	14.9	0.7998	80.0	0.7879	-0.0120	0.7613	0.0311
					90.0	0.7774	-0.0225	0.7397	0.0312
					95.0	0.7702	-0.0297	0.7200	0.0312

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Confidence Limit			Estimate Percentile Values			
				Minimum	10 %	25 %	Median	75 %	90 %	Maximum
06a	0.79984	0	.80	.65710	.72329	.74237	.76278	.78275	.80208	.86406
			.90	.63521	.70147	.72061	.74120	.76129	.78104	.84358
			.95	.61595	.68192	.70099	.72156	.74159	.76154	.82429
06b	0.79984	0	.80	.64967	.72275	.74021	.76058	.78276	.80405	.84749
			.90	.62777	.70090	.71848	.73890	.76130	.78282	.82723
			.95	.60855	.68131	.69886	.71927	.74161	.76317	.80793
06c	0.79984	0	.80	.65784	.72172	.74177	.76064	.78345	.79871	.85272
			.90	.63582	.69981	.71998	.73895	.76205	.77736	.83260
			.95	.61643	.68017	.70032	.71931	.74241	.75778	.81336

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	RS	CL	\hat{R}^*	D	Mean	S.D.
07a	32	4.67	3.03	0.7917	80.0	0.7557	-0.0359	0.6850	0.0807
					90.0	0.7012	-0.0904	0.6144	0.0715
					95.0	0.6202	-0.1715	0.5406	0.0578
07b	32	4.67	3.03	0.7917	80.0	0.7627	-0.0290	0.6883	0.0811
					90.0	0.7034	-0.0883	0.6176	0.0716
					95.0	0.6254	-0.1663	0.5436	0.0572
07c	32	4.67	3.03	0.7917	80.0	0.7503	-0.0414	0.6844	0.0773
					90.0	0.6996	-0.0921	0.6144	0.0687
					95.0	0.6202	-0.1715	0.5419	0.0566

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Minimum	Confidence 10 %	Limit 25 %	Estimate Median	Percentile Values		
								75 %	90 %	Maximum
07a	0.79167	0	.80 .90 .95	.44802 .39723 .35308	.58087 .52024 .46297	.63122 .56678 .50207	.68742 .61866 .54601	.73980 .66591 .58169	.79090 .70123 .60634	.95813 .91337 .83005
07b	0.79167	0	.80 .90 .95	.45284 .40095 .35576	.57832 .51803 .46283	.63451 .57087 .50859	.69260 .62358 .55297	.74744 .67360 .58580	.79082 .70336 .61029	.94593 .88892 .78501
07c	0.79167	0	.80 .90 .95	.38869 .34237 .30297	.58239 .52236 .46551	.63401 .56991 .50816	.68801 .61936 .54743	.73831 .66531 .58297	.78287 .69960 .60634	.97779 .95352 .90680

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	RS	CL	\hat{R}^*	D	Mean	S.D.
08a	32	11.7	7.57	0.7917	80.0	0.7788	-0.0129	0.7368	0.0520
					90.0	0.7689	-0.0228	0.7026	0.0518
					95.0	0.7528	-0.0389	0.6702	0.0504
08b	32	11.7	7.57	0.7917	80.0	0.7777	-0.0140	0.7331	0.0541
					90.0	0.7694	-0.0222	0.6989	0.0537
					95.0	0.7551	-0.0365	0.6666	0.0524
08c	32	11.7	7.57	0.7917	80.0	0.7767	-0.0150	0.7327	0.0530
					90.0	0.7632	-0.0285	0.6986	0.0526
					95.0	0.7487	-0.0429	0.6663	0.0512

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Minimum	Confidence 10 %	Limit 25 %	Estimate Median	Percentile Values		
								75 %	90 %	Maximum
08a	0.79167	0	.80 .90 .95	.58162 .54914 .52041	.67194 .63828 .60763	.69966 .66556 .63410	.73759 .70304 .67082	.76997 .73573 .70297	.80417 .76888 .73495	.90349 .86814 .82644
08b	0.79167	0	.80 .90 .95	.57794 .54570 .51722	.66379 .63015 .59962	.69643 .66267 .63157	.73256 .69836 .66632	.76951 .73545 .70212	.80434 .76942 .73532	.89135 .85627 .81640
08c	0.79167	0	.80 .90 .95	.58949 .55696 .52813	.66094 .62707 .59640	.69694 .66308 .63194	.73291 .69875 .66673	.76725 .73315 .70036	.79764 .76332 .72952	.91141 .86608 .81783

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	RS	CL	\hat{R}^*	D	Mean	S.D.
09a	32	23.4	15.1	0.7917	80.0	0.7871	-0.0046	0.7551	0.0383
					90.0	0.7817	-0.0100	0.7335	0.0386
					95.0	0.7764	-0.0152	0.7139	0.0385
09b	32	23.4	15.1	0.7917	80.0	0.7857	-0.0060	0.7542	0.0385
					90.0	0.7813	-0.0104	0.7326	0.0388
					95.0	0.7742	-0.0175	0.7130	0.0388
09c	32	23.4	15.1	0.7917	80.0	0.7839	-0.0078	0.7538	0.0383
					90.0	0.7819	-0.0098	0.7322	0.0385
					95.0	0.7755	-0.0162	0.7125	0.0385

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Confidence Limit		Estimate	Percentile Values			
				Minimum	10 %		25 %	Median	75 %	90 %
09a	0.79167	0	.80	.63478	.70642	.72995	.75499	.78073	.80272	.86981
			.90	.61288	.68446	.70810	.73337	.75940	.78166	.84982
			.95	.59372	.66487	.68846	.71376	.73958	.76216	.83037
09b	0.79167	0	.80	.62609	.70625	.72913	.75489	.77990	.80230	.88091
			.90	.60430	.68427	.70722	.73326	.75852	.78128	.86134
			.95	.58526	.66467	.68754	.71365	.73892	.76175	.84203
09c	0.79167	0	.80	.61157	.70578	.72972	.75441	.77767	.80320	.86563
			.90	.58980	.68377	.70785	.73274	.75610	.78190	.84564
			.95	.57085	.66415	.68820	.71310	.73634	.76221	.82627

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	\hat{R}^*o	D	Mean	S.D.
10a	32	6.59	4.27	0.7232	80.0	0.6889	-0.0343	0.6233	0.0758
					90.0	0.6471	-0.0761	0.5597	0.0697
					95.0	0.5932	-0.1300	0.4983	0.0606
10b	32	6.59	4.27	0.7232	80.0	0.6897	-0.0335	0.6227	0.0800
					90.0	0.6544	-0.0688	0.5592	0.0736
					95.0	0.5985	-0.1247	0.4976	0.0636
10c	32	6.59	4.27	0.7232	80.0	0.6906	-0.0326	0.6340	0.0780
					90.0	0.6520	-0.0712	0.5602	0.0717
					95.0	0.5932	-0.1299	0.4984	0.0625

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Confidence Limit			Estimate Percentile Values			
				Lower Minimum	10 %	25 %	Median	75 %	90 %	Maximum
10a	0.72317	0	.80	.38169	.52320	.56765	.62362	.67774	.71491	.82429
			.90	.33622	.46646	.50850	.56045	.61015	.64708	.74116
			.95	.29763	.41592	.45525	.49977	.54475	.57482	.63872
10b	0.72317	0	.80	.40848	.51967	.56637	.61984	.67426	.72627	.88751
			.90	.36039	.46321	.50735	.55732	.60872	.65438	.77661
			.95	.31920	.41270	.45360	.49868	.54328	.57726	.65117
10c	0.72317	0	.80	.41254	.51980	.56890	.62469	.68110	.72407	.84018
			.90	.36448	.46295	.50943	.56162	.61399	.65197	.74113
			.95	.32326	.41345	.45571	.50127	.54580	.57756	.64522

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	\hat{R}^*	D	Mean	S. D.
11a	32	16.5	10.7	0.7232	80.0	0.7099	-0.0132	0.6647	0.0520
					90.0	0.6953	-0.0279	0.6311	0.0514
					95.0	0.6779	-0.0453	0.6004	0.0502
11b	32	16.5	10.7	0.7232	80.0	0.7060	-0.0171	0.6636	0.0510
					90.0	0.6938	-0.0294	0.6300	0.0505
					95.0	0.6817	-0.0415	0.5994	0.0493
11c	32	16.5	10.7	0.7232	80.0	0.7115	-0.0116	0.6672	0.0525
					90.0	0.7011	-0.0220	0.6335	0.0520
					95.0	0.6843	-0.0389	0.6028	0.0508

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Minimum	Confidence 10 %	Limit 25 %	Estimate Median	Percentile Values		
								75 %	90 %	Maximum
11a	0.72317	0	.80 .90 .95	.47279 .44301 .41741	.59689 .56430 .53534	.62798 .59476 .56494	.66587 .63212 .60167	.70222 .66834 .63701	.72980 .69533 .66304	.82596 .79100 .75559
11b	0.72317	0	.80 .90 .95	.48277 .45267 .42673	.59915 .56623 .53695	.63033 .59708 .56710	.66350 .62984 .59929	.69693 .66292 .63184	.72775 .69383 .66190	.84057 .80645 .77143
11c	0.72317	0	.80 .90 .95	.52162 .49048 .46339	.59916 .56624 .53696	.63076 .59752 .56769	.66657 .63266 .60180	.70357 .66908 .63763	.73559 .70114 .66848	.82379 .78845 .75267

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	\hat{R}^*	D	Mean	S. D.
12a	32	32.9	21.4	0.7232	80.0	0.7151	-0.0081	0.6849	0.0358
					90.0	0.7096	-0.0136	0.6630	0.0358
					95.0	0.7019	-0.0213	0.6435	0.0356
12b	32	32.9	21.4	0.7232	80.0	0.7147	-0.0085	0.6845	0.0368
					90.0	0.7083	-0.0149	0.6627	0.0367
					95.0	0.7003	-0.0229	0.6433	0.0366
12c	32	32.9	21.4	0.7232	80.0	0.7165	-0.0067	0.6868	0.0362
					90.0	0.7120	-0.0112	0.6649	0.0363
					95.0	0.7045	-0.0187	0.6455	0.0360

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Minimum	Confidence 10 %	Limit 25 %	Estimate Median	Percentile 75 %	90 %	Maximum Values
12a	0.72317	0	.80	.59098	.63998	.66109	.68369	.70760	.73135	.80584
			.90	.56944	.61814	.63921	.66167	.68570	.70958	.78478
			.95	.55076	.59901	.61990	.64216	.66617	.69002	.76528
12b	0.72317	0	.80	.55970	.63851	.66109	.68655	.71031	.73015	.78661
			.90	.53843	.61662	.63915	.66439	.68836	.70831	.76518
			.95	.52009	.59744	.61975	.64485	.66876	.68868	.74550
12c	0.72317	0	.80	.58726	.64023	.66333	.68675	.70933	.73387	.79268
			.90	.56573	.61830	.64138	.66470	.68732	.71203	.77146
			.95	.54707	.59904	.62205	.64515	.66768	.69238	.75189

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	RS	CL	R*o	D	Mean	S.D.
13a	19	15.4	10.7	0.7233	80.0	0.7195	-0.0038	0.6658	0.0647
					90.0	0.7176	-0.0057	0.6335	0.0662
					95.0	0.7119	-0.0114	0.6042	0.0672
13b	19	15.4	10.7	0.7233	80.0	0.7198	-0.0035	0.6676	0.0622
					90.0	0.7143	-0.0090	0.6354	0.0637
					95.0	0.7123	-0.0110	0.6060	0.0649
13c	19	15.4	10.7	0.7233	80.0	0.7263	+0.0030	0.6696	0.0680
					90.0	0.7264	+0.0031	0.6373	0.0696
					95.0	0.7257	+0.0024	0.6080	0.0707

Table D-1. Simulation Results (continued).

Case	RS	Null Runs	CL	Lower Minimum	Confidence 10 %	Limit 25 %	Estimate Median	Percentile 75 %	Values 90 %	Maximum
13a	0.72329	0	.80	.48075	.57589	.62474	.66831	.70837	.74695	.85199
			.90	.44732	.54331	.59298	.63561	.67681	.71758	.82642
			.95	.41620	.51302	.56258	.60410	.64881	.69143	.80113
13b	0.72329	0	.80	.46207	.58839	.62658	.67098	.71049	.74397	.85327
			.90	.42661	.55217	.59377	.63913	.67878	.71427	.82692
			.95	.39601	.52107	.56404	.60892	.65022	.68551	.80068
13c	0.72329	0	.80	.42256	.58572	.62423	.67044	.71730	.75410	.85472
			.90	.38883	.55042	.58986	.63877	.68509	.72639	.82958
			.95	.36002	.51810	.55940	.60920	.65589	.70087	.80469

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	RS	CL	\hat{R}^*	D	Mean	S.D.
14a	15	4.40	3.14	0.7924	80.0	0.7561	-0.0363	0.6903	0.0758
					90.0	0.6865	-0.1059	0.6163	0.0647
					95.0	0.6388	-0.1536	0.5375	0.0652
14b	15	4.40	3.14	0.7924	80.0	0.7561	-0.0363	0.6898	0.0776
					90.0	0.6865	-0.1059	0.6162	0.0669
					95.0	0.6388	-0.1536	0.5379	0.0665
14c	15	4.40	3.14	0.7924	80.0	0.7561	-0.0363	0.6890	0.0744
					90.0	0.6865	-0.1059	0.6162	0.0642
					95.0	0.6388	-0.1536	0.5394	0.0633

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Minimum	Confidence 10 %	Limit 25 %	Estimate Median	Percentile Values		
								75 %	90 %	Maximum
14a	0.79239	5	.80	.43820	.58497	.62901	.69696	.75814	.79927	.80327
				.38840	.52379	.56639	.62751	.67696	.72709	
				.34527	.44252	.49880	.55675	.57773	.61078	
14b	0.79239	9	.80	.40335	.58721	.62901	.70586	.75814	.79927	.80327
				.35524	.52621	.56639	.62751	.67696	.72709	
				.31410	.44252	.49880	.55675	.57773	.61078	
14c	0.79239	10	.80	.42400	.58497	.62901	.69696	.74888	.79927	.80327
				.37505	.52379	.56639	.62751	.66470	.72709	
				.33289	.45711	.50391	.55675	.57659	.61078	

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	\hat{R}^*	D	Mean	S.D.
15a	25	3.12	2.09	0.8822	80.0	0.8236	-0.0586	0.7790	0.0535
					90.0	0.7402	-0.1420	0.6969	0.0392
					95.0	0.6455	-0.2367	0.5962	0.0696
15b	25	3.12	2.09	0.8822	80.0	0.8236	-0.0586	0.7795	0.0520
					90.0	0.7402	-0.1420	0.6997	0.0394
					95.0	0.6455	-0.2367	0.6030	0.0653
15c	25	3.12	2.09	0.8822	80.0	0.8236	-0.0586	0.7789	0.0536
					90.0	0.7402	-0.1420	0.6991	0.0407
					95.0	0.6455	-0.2367	0.6025	0.0651

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Minimum	Confidence 10 %	Limit 25 %	Estimate Median	Percentile Values									
								75 %	90 %	Maximum							
15a	0.88222	40	.80	.57786	.69497	.75679	.79095	.82358	.83590	.84003							
											.90	.64006	.68406	.69292	.72127	.74017	.77600
15b	0.88222	53	.80	.55247	.69497	.75679	.79095	.82358	.83590	.84003							
											.90	.50185	.64006	.68406	.69292	.74017	.77600
15c	0.88222	45	.80	.57525	.69497	.75679	.79095	.82358	.83590	.84003							
											.90	.52461	.64006	.68406	.70531	.74017	.77600

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	RS	CL	\hat{R}^*	D	Mean	S.D.
16a	25	6.25	4.18	0.8822	80.0	0.8699	-0.0123	0.8285	0.0457
					90.0	0.8493	-0.0329	0.7923	0.0432
					95.0	0.8023	-0.0799	0.7540	0.0388
16b	25	6.25	4.18	0.8822	80.0	0.8699	-0.0123	0.8277	0.0461
					90.0	0.8493	-0.0329	0.7916	0.0437
					95.0	0.8023	-0.0799	0.7534	0.0392
16c	25	6.25	4.18	0.8822	80.0	0.8699	-0.0123	0.8278	0.0452
					90.0	0.8493	-0.0329	0.7914	0.0425
					95.0	0.8023	-0.0799	0.7527	0.0386

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Minimum	Confidence 10 %	Limit 25 %	Estimate Median	Percentile Values		
								75 %	90 %	Maximum
16a	0.88222	0	.80	.66422	.77665	.79675	.83156	.86994	.88935	.91737
				.63061	.74214	.76304	.79577	.82708	.84928	.88207
				.60010	.69549	.72955	.75911	.78548	.80232	.83826
16b	0.88222	2	.80	.66560	.77722	.79675	.83156	.85381	.88935	.91737
				.63208	.74214	.76304	.79577	.82032	.84928	.88207
				.60165	.69549	.72955	.75911	.78548	.80232	.83826
16c	0.88222	4	.80	.69530	.76147	.79629	.83156	.85381	.88935	.91428
				.66153	.72767	.76252	.79577	.82032	.84928	.86134
				.63046	.69498	.72897	.75911	.78548	.79841	.82273

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	RS	CL	\hat{R}^*	D	Mean	S.D.
17a	25	12.5	8.36	0.8822	80.0	0.8816	-0.0006	0.8493	0.0341
					90.0	0.8716	-0.0107	0.8291	0.0346
					95.0	0.8614	-0.0208	0.8098	0.0343
17b	25	12.5	8.36	0.8822	80.0	0.8728	-0.0095	0.8479	0.0338
					90.0	0.8716	-0.0107	0.8277	0.0344
					95.0	0.8650	-0.0172	0.8084	0.0341
17c	25	12.5	8.36	0.8822	80.0	0.8816	-0.0006	0.8508	0.0334
					90.0	0.8735	-0.0087	0.8308	0.0340
					95.0	0.8614	-0.0208	0.8113	0.0336

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Minimum	Confidence Limit 10 %	Estimate Median	Percentile 75 %	Percentile 90 %	Values Maximum
17a	0.88222	0	.80	.72335	.80692	.85305	.87263	.89156	.94305
			.90	.70154	.78597	.83297	.85304	.87156	.92156
			.95	.68198	.76656	.81378	.83397	.85146	.89572
17c	0.88222	0	.80	.72300	.80702	.84394	.87263	.89156	.93270
			.90	.70116	.78608	.82371	.85304	.87156	.91237
			.95	.68159	.76668	.80451	.83397	.85146	.88947
17c	0.88222	0	.80	.73138	.80702	.85328	.87263	.89261	.93270
			.90	.70962	.78609	.83323	.85304	.87353	.91237
			.95	.69006	.76668	.81406	.83397	.85448	.88947

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	RS	CL	\hat{R}^*	D	Mean	S.D.
18a	25	0.62	0.42	0.9753	80.0	0.8359	-0.1394	0.8305	0.0195
					90.0	0.7402	-0.2351	0.6993	0.0277
					95.0	0.6437	-0.3316	0.5104	0.0836
18b	25	0.62	0.42	0.9753	80.0	0.8359	-0.1394	0.8306	0.0200
					90.0	0.7402	-0.2351	0.6972	0.0263
					95.0	0.6437	-0.3316	0.5044	0.0804
18c	25	0.62	0.42	0.9753	80.0	0.8359	-0.1394	0.8304	0.0198
					90.0	0.7402	-0.2351	0.6981	0.0267
					95.0	0.6437	-0.3316	0.5073	0.0817

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Minimum	Confidence 10 %	Limit 25 %	Estimate Median	Percentile Values		
								75 %	90 %	Maximum
18a	0.97530	1184	.80 .90 .95	.72345 .66288 .45815	.82358 .68406 .45815	.82358 .68406 .45815	.83590 .68406 .45815	.83590 .72127 .63745	.83590 .74017 .63745	.84003 .77600 .70008
18b	0.97530	1164	.80 .90 .95	.72802 .67179 .45815	.82358 .68406 .45815	.83590 .68406 .45815	.83590 .68406 .45815	.83590 .69292 .62593	.83590 .74017 .63745	.84003 .77600 .70008
18c	0.97530	1251	.80 .90 .95	.75679 .68406 .45815	.82358 .68406 .45815	.83590 .68406 .45815	.83590 .68406 .45815	.83590 .72127 .63745	.83590 .74017 .63745	.84003 .77600 .70008

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	RS	CL	\hat{R}^*	D	Mean	S.D.
19a	25	1.25	0.84	0.9753	80.0	0.9143	-0.0610	0.9069	0.0156
					90.0	0.8603	-0.1150	0.8409	0.0194
					95.0	0.8023	-0.1730	0.7372	0.0622
19b	25	1.25	0.84	0.9753	80.0	0.9143	-0.0610	0.9067	0.0163
					90.0	0.8603	-0.1150	0.8402	0.0193
					95.0	0.8023	-0.1730	0.7352	0.0618
19c	25	1.25	0.84	0.9753	80.0	0.9143	-0.0610	0.9064	0.0167
					90.0	0.8603	-0.1150	0.8396	0.0191
					95.0	0.8023	-0.1730	0.7342	0.0615

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Minimum	Confidence 10 %	Limit 25 %	Estimate Median	Percentile Values		
								75 %	90 %	Maximum
19a	0.97530	427	.80 .90 .95	:83156 :79577 :67687	:88935 :82708 :67687	:90751 :82708 :67687	:91428 :82708 :67687	:91428 :86033 :79841	:91428 :86033 :80232	:91737 :88207 :83826
19b	0.97530	424	.80 .90 .95	:81518 :78157 :67687	:88935 :82708 :67687	:90751 :82708 :67687	:91428 :82708 :67687	:91428 :86033 :79841	:91428 :86033 :80232	:91737 :88207 :83826
19c	0.97530	438	.80 .90 .95	:81299 :77752 :67687	:88935 :82708 :67687	:90751 :82708 :67687	:91428 :82708 :67687	:91428 :86033 :79841	:91428 :86033 :80232	:91737 :88207 :83826

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	\hat{R}^*	D	Mean	S.D.
20a	25	2.50	1.67	0.9753	80.0	0.9562	-0.0191	0.9439	0.0181
					90.0	0.9275	-0.0478	0.9168	0.0102
					95.0	0.8971	-0.0782	0.8770	0.0322
20b	25	2.50	1.67	0.9753	80.0	0.9562	-0.0191	0.9438	0.0182
					90.0	0.9275	-0.0478	0.9165	0.0098
					95.0	0.8971	-0.0782	0.8764	0.0324
20c	25	2.50	1.67	0.9753	80.0	0.9562	-0.0191	0.9436	0.0184
					90.0	0.9275	-0.0478	0.9164	0.0099
					95.0	0.8971	-0.0782	0.8763	0.0321

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Minimum	Confidence 10 %	Limit 25 %	Estimate Median	Percentile Values		
								75 %	90 %	Maximum
20a	0.97530	89	.80	.88240	.92226	.93511	.94636	.95263	.95618	.95800
			.90	.86306	.90232	.90944	.92156	.92754	.93949	
			.95	.82272	.82272	.87538	.89354	.89572	.91598	
20b	0.97530	96	.80	.88265	.92402	.93270	.94636	.95263	.95618	.95800
			.90	.86334	.90572	.90944	.91705	.92754	.93949	
			.95	.82272	.82272	.87127	.89354	.89572	.91598	
20c	0.97530	102	.80	.88265	.92373	.93511	.94636	.95263	.95618	.95800
			.90	.86334	.90563	.90944	.91705	.92754	.93949	
			.95	.82272	.82272	.87127	.89354	.89572	.91598	

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	RS	CL	\hat{R}^*	D	Mean	S.D.
21a	25	5.00	3.34	0.9753	80.0	0.9711	-0.0042	0.9603	0.0158
					90.0	0.9631	-0.0122	0.9493	0.0137
					95.0	0.9472	-0.0281	0.9363	0.0126
21b	25	5.00	3.34	0.9753	80.0	0.9711	-0.0042	0.9607	0.0157
					90.0	0.9600	-0.0153	0.9496	0.0136
					95.0	0.9472	-0.0281	0.9362	0.0128
21c	25	5.00	3.34	0.9753	80.0	0.9711	-0.0122	0.9492	0.0141
					90.0	0.9631	-0.0122	0.9492	0.0141
					95.0	0.9472	-0.0281	0.9360	0.0125

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Confidence Limit		Estimate		Percentile Values	
				Minimum	10 %	25 %	Median	75 %	90 %
21a	0.97530	8	.80 .90 .95	.91857 .90748 .89683	.94488 .93475 .91889	.95494 .94449 .93019	.96034 .94990 .93849	.97111 .95768 .94527	.97603 .96309 .94643
21b	0.97530	5	.80 .90 .95	.91860 .90752 .89687	.94488 .93475 .91889	.95494 .94449 .93019	.96138 .95183 .93849	.97111 .95768 .94527	.97603 .95998 .94643
21c	0.97530	3	.80 .90 .95	.91349 .90223 .89149	.94485 .93471 .91889	.95030 .94037 .93019	.96034 .94990 .93849	.97111 .95768 .94527	.97603 .96309 .94643

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	RS	CL	\hat{R}^*	D	Mean	S. D.
22a	60	2.70	1.62	0.8731	80.0	0.8262	-0.0469	0.7790	0.0679
					90.0	0.7512	-0.1219	0.6943	0.0634
					95.0	0.6521	-0.2209	0.5911	0.0660
22b	60	2.70	1.62	0.8731	80.0	0.8288	-0.0443	0.7811	0.0680
					90.0	0.7525	-0.1206	0.6953	0.0625
					95.0	0.6530	-0.2201	0.5899	0.0642
22c	60	2.70	1.62	0.8731	80.0	0.8246	-0.0484	0.7761	0.0676
					90.0	0.7478	-0.1253	0.6909	0.0616
					95.0	0.6495	-0.2236	0.5873	0.0636

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Confidence Limit Estimate Percentile Values						
				Lower Minimum	10 %	25 %	Median	75 %	90 %	Maximum
22a	0.87306	0	.80 .90 .95	.52180 .46500 .37713	.69050 .62229 .52792	.73853 .66195 .56676	.78167 .69235 .59199	.81674 .72130 .61670	.85286 .75116 .63962	.99093 .98088 .96110
22b	0.87306	0	.80 .90 .95	.55336 .49445 .37971	.69440 .62650 .52699	.73977 .66268 .56710	.78298 .69292 .59095	.82177 .72486 .61793	.85451 .75247 .63880	.99474 .98890 .97731
22c	0.87306	0	.80 .90 .95	.52401 .46732 .37810	.69267 .62483 .52751	.73345 .65758 .56343	.78012 .69134 .58907	.81573 .72028 .61514	.84941 .74775 .63736	.99585 .99124 .98207

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	RS	CL	\hat{R}^*	D	Mean	S. D.
23a	60	3.37	2.02	0.8731	80.0	0.8410	-0.0321	0.7942	0.0570
					90.0	0.7785	-0.0945	0.7255	0.0499
					95.0	0.7003	-0.1728	0.6442	0.0464
23b	60	3.37	2.02	0.8731	80.0	0.8419	-0.0312	0.7949	0.0616
					90.0	0.7825	-0.0906	0.7270	0.0560
					95.0	0.7029	-0.1701	0.6467	0.0514
23c	60	3.37	2.02	0.8731	80.0	0.8391	-0.0340	0.7926	0.0593
					90.0	0.7767	-0.0964	0.7242	0.0523
					95.0	0.7001	-0.1729	0.6434	0.0471

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Confidence Limit Estimate Percentile Values						
				10 %	25 %	75 %				
23a	0.87306	0	.80	.59988	.72351	.75778	.79763	.83538	.85795	.97649
			.90	.54795	.66462	.69580	.72965	.75616	.77854	.95084
			.95	.45881	.59143	.62638	.65049	.67132	.69025	.90157
23b	0.87306	0	.80	.59403	.72129	.75771	.79692	.83623	.86128	.98131
			.90	.54167	.66402	.69603	.73001	.75780	.78248	.96081
			.95	.46142	.59407	.62577	.65024	.67127	.69244	.92110
23c	0.87306	0	.80	.62630	.71817	.75536	.79716	.83404	.85771	.96837
			.90	.57296	.66041	.69307	.73122	.75588	.77669	.93418
			.95	.46050	.58928	.62165	.64834	.67042	.68987	.86939

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	RS	CL	\hat{R}^*	D	Mean	S.D.
24a	60	6.75	4.04	0.8731	80.0	0.8640	-0.0091	0.8260	0.0442
					90.0	0.8445	-0.0286	0.7910	0.0433
					95.0	0.8118	-0.0613	0.7548	0.0402
24b	60	6.75	4.04	0.8731	80.0	0.8619	-0.0112	0.8239	0.0437
					90.0	0.8425	-0.0305	0.7890	0.0429
					95.0	0.8122	-0.0608	0.7531	0.0402
24c	60	6.75	4.04	0.8731	80.0	0.8573	-0.0158	0.8194	0.0445
					90.0	0.8408	-0.0323	0.7846	0.0439
					95.0	0.8090	-0.0641	0.7491	0.0413

Table D-1. Simulation Results (continued).

Case	RS	Null Runs	CL	Lower Minimum	Confidence 10 %	Limit 25 %	Estimate Median	Percentile Values		
								75 %	90 %	Maximum
24a	0.87306	0	.80 .90 .95	.66551 .63138 .60036	.77050 .73626 .70318	.79687 .76276 .72856	.82778 .79314 .75803	.85871 .82338 .78587	.88109 .84450 .80365	.93288 .88911 .89870
24b	0.87306	0	.80 .90 .95	.64083 .60736 .57719	.76983 .73544 .70207	.79571 .76107 .72747	.82485 .79011 .75513	.85311 .81862 .78299	.87970 .84252 .80259	.93627 .89290 .84319
24c	0.87306	0	.80 .90 .95	.66632 .63226 .60128	.76170 .72751 .69529	.79211 .75776 .72426	.82243 .78764 .75260	.84971 .81534 .77863	.87603 .84081 .80057	.93976 .89862 .83745

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	RS	CL	\hat{R}^*	D	Mean	S.D.
25a	60	13.5	8.09	0.8731	80.0	0.8679	-0.0052	0.8411	0.0324
					90.0	0.8609	-0.0122	0.8206	0.0328
					95.0	0.8519	-0.0212	0.8011	0.0328
25b	60	13.5	8.09	0.8731	80.0	0.8693	-0.0037	0.8426	0.0317
					90.0	0.8622	-0.0109	0.8221	0.0322
					95.0	0.8524	-0.0206	0.8026	0.0321
25c	60	13.5	8.09	0.8731	80.0	0.8678	-0.0053	0.8411	0.0331
					90.0	0.8616	-0.0115	0.8206	0.0335
					95.0	0.8529	-0.0202	0.8011	0.0333

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Confidence Limit			Estimate Percentile Values			
				Minimum	10 %	25 %	Median	75 %	90 %	Maximum
25a	0.87306	0	.80 .90 .95	.73071 .70890 .68930	.79953 .77826 .75857	.82120 .80022 .78082	.84188 .82141 .80200	.86174 .84177 .82235	.88054 .86092 .84155	.92451 .90512 .88415
25b	0.87306	0	.80 .90 .95	.71861 .69672 .67713	.80235 .78122 .76145	.82333 .80252 .78301	.84342 .82297 .80349	.86329 .84318 .82390	.88198 .86215 .84271	.92267 .90405 .88429
25c	0.87306	0	.80 .90 .95	.73682 .71493 .69522	.79911 .77780 .75818	.81992 .79914 .77971	.84246 .82207 .80271	.86295 .84275 .82339	.88116 .86162 .84207	.92694 .90814 .88779

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	RS	CL	R*o	D	Mean	S.D.
26a	60	3.58	2.14	0.8329	80.0	0.7999	-0.0330	0.7462	0.0671
					90.0	0.7262	-0.1067	0.6670	0.0580
					95.0	0.6341	-0.1988	0.5770	0.0510
26b	60	3.58	2.14	0.8329	80.0	0.8013	-0.0317	0.7480	0.0657
					90.0	0.7259	-0.1070	0.6687	0.0579
					95.0	0.6388	-0.1941	0.5786	0.0542
26c	60	3.58	2.14	0.8329	80.0	0.8016	-0.0313	0.7479	0.0683
					90.0	0.7291	-0.1038	0.6689	0.0606
					95.0	0.6389	-0.1941	0.5790	0.0550

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Minimum	Confidence 10 %	Limit 25 %	Estimate Median	Percentile Values		
								75 %	90 %	Maximum
26a	0.83293	0	.80 .90 .95	.50494 .44907 .38278	.65423 .58884 .51215	.70476 .63339 .55374	.75277 .67574 .58451	.79207 .70396 .60620	.82132 .72624 .62661	.99003 .97899 .95730
26b	0.83293	0	.80 .90 .95	.44914 .39837 .35423	.66803 .60232 .52336	.70691 .63512 .55745	.75246 .67374 .58320	.79448 .70225 .60475	.81974 .72594 .62804	.99182 .98276 .96488
26c	0.83293	0	.80 .90 .95	.49078 .43589 .38318	.65833 .59225 .52008	.70811 .63602 .55696	.75293 .67552 .58450	.79397 .70268 .60797	.82203 .72909 .62949	.99675 .99313 .98594

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	\hat{R}^*	D	Mean	S.D.
27a	60	4.47	2.68	0.8329	80.0	0.8104	-0.0226	0.7572	0.0615
					90.0	0.7577	-0.0752	0.6938	0.0546
					95.0	0.6870	-0.1459	0.6246	0.0459
27b	60	4.47	2.68	0.8329	80.0	0.8106	-0.0223	0.7589	0.0606
					90.0	0.7543	-0.0786	0.6954	0.0535
					95.0	0.6848	-0.1482	0.6257	0.0452
27c	60	4.47	2.68	0.8329	80.0	0.8126	-0.0203	0.7591	0.0624
					90.0	0.7601	-0.0729	0.6962	0.0565
					95.0	0.6901	-0.1428	0.6278	0.0478

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Minimum	Confidence 10 %	Limit 25 %	Estimate Median	Percentile Values		
								75 %	90 %	Maximum
27a	0.83293	0	.80 .90 .95	.50452 .45751 .41601	.67785 .62261 .56529	.71738 .65951 .59974	.75978 .69857 .63164	.79907 .73381 .65610	.83384 .75769 .67397	.94963 .89629 .79846
27b	0.83293	0	.80 .90 .95	.51810 .46947 .42629	.67842 .62258 .56412	.72067 .66315 .60144	.76371 .70358 .63689	.79991 .73528 .65631	.83328 .75428 .67342	.94105 .87922 .76751
27c	0.83293	0	.80 .90 .95	.54827 .49872 .45421	.67671 .61950 .56337	.72177 .66378 .60268	.76238 .70030 .63476	.80267 .73596 .65904	.83756 .76007 .67770	.97651 .95089 .90167

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	\hat{R}^*	D	Mean	S.D.
28a	60	8.94	5.36	0.8329	80.0	0.8221	-0.0108	0.7846	0.0459
					90.0	0.8066	-0.0263	0.7502	0.0456
					95.0	0.7856	-0.0473	0.7165	0.0439
28b	60	8.94	5.36	0.8329	80.0	0.8249	-0.0080	0.7854	0.0478
					90.0	0.8092	-0.0237	0.7510	0.0477
					95.0	0.7911	-0.0419	0.7172	0.0459
28c	60	8.94	5.36	0.8329	80.0	0.8251	-0.0078	0.7832	0.0473
					90.0	0.8080	-0.0249	0.7487	0.0470
					95.0	0.7845	-0.0484	0.7150	0.0453

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Confidence Limit			Estimate Percentile Values			
				Lower Minimum	10 %	25 %	Median	75 %	90 %	Maximum
28a	0.83293	0	.80 .90 .95	.63064 .59758 .56788	.72377 .68954 .65760	.75268 .71818 .68544	.78790 .75363 .72030	.81578 .78143 .74695	.84218 .80659 .77134	.94019 .89934 .83842
28b	0.83293	0	.80 .90 .95	.62536 .59199 .56204	.71988 .68591 .65447	.75456 .71996 .68714	.78770 .75361 .72032	.81720 .78241 .74817	.84323 .80921 .77337	.91227 .87495 .82952
28c	0.83293	0	.80 .90 .95	.62536 .59200 .56205	.72284 .68852 .65650	.75191 .71714 .68452	.78245 .74805 .71466	.81647 .78204 .74790	.84227 .80799 .77207	.91056 .87266 .82558

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	\hat{R}^*	D	Mean	S.D.
29a	60	17.9	10.7	0.8329	80.0	0.8269	-0.0060	0.8002	0.0326
					90.0	0.8198	-0.0131	0.7791	0.0329
					95.0	0.8111	-0.0219	0.7595	0.0329
29b	60	17.9	10.7	0.8329	80.0	0.8288	-0.0041	0.8002	0.0339
					90.0	0.8230	-0.0099	0.7791	0.0324
					95.0	0.8137	-0.0192	0.7595	0.0343
29c	60	17.9	10.7	0.8329	80.0	0.8299	-0.0030	0.8016	0.0344
					90.0	0.8227	-0.0103	0.7805	0.0348
					95.0	0.8156	-0.0173	0.7609	0.0348

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Minimum	Confidence 10 %	Limit 25 %	Estimate Median	Percentile Values		
								75 %	90 %	Maximum
29a	0.83293	0	.80 .90 .95	.67933 .65728 .63776	.75979 .73825 .71851	.78117 .75985 .74026	.80152 .78054 .76102	.82098 .80016 .78049	.84042 .81979 .80022	.90200 .88303 .86379
29b	0.83293	0	.80 .90 .95	.67260 .65057 .63112	.75726 .73553 .71581	.77803 .75649 .73675	.80023 .77902 .75938	.82430 .80359 .78410	.84348 .82301 .80374	.88634 .86695 .84770
29c	0.83293	0	.80 .90 .95	.70176 .67973 .66021	.75812 .73646 .71679	.77888 .75741 .73774	.80255 .78138 .76162	.82497 .80425 .78484	.84331 .82268 .80304	.89342 .87374 .85396

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	\hat{R}^*	D	Mean	S.D.
30a	25	0.62	0.42	0.9753	80.0	0.9377	-0.0376	0.8894	0.0554
					90.0	0.9120	-0.0633	0.8156	0.1078
					95.0	0.8871	-0.0882	0.7159	0.1969
30b	25	0.62	0.42	0.9753	80.0	0.9377	-0.0376	0.8895	0.0554
					90.0	0.9120	-0.0633	0.8152	0.1083
					95.0	0.8871	-0.0882	0.7144	0.1983
30c	25	0.62	0.42	0.9753	80.0	0.9377	-0.0376	0.8917	0.0551
					90.0	0.9120	-0.0633	0.8203	0.1071
					95.0	0.8871	-0.0882	0.7243	0.1954

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Minimum	Confidence Limit 25 %	Estimate Median	Percentile 75 %	Percentile Values	
								90 %	Maximum
30a	0.97530	549	.80	.72345	.82358	.83590	.93765	.93765	.93765
			.90	.66288	.68406	.68406	.91201	.91201	.91201
			.95	.45815	.45815	.45815	.88707	.88707	.88707
30b	0.97530	550	.80	.75679	.82358	.83590	.93765	.93765	.93765
			.90	.68406	.68406	.68406	.91201	.91201	.91201
			.95	.45815	.45815	.45815	.88707	.88707	.88707
30c	0.97530	571	.80	.75679	.82358	.83590	.93765	.93765	.93765
			.90	.68406	.68406	.68406	.91201	.91201	.91201
			.95	.45815	.45815	.45815	.88707	.88707	.88707

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	RS	CL	\hat{R}^*	D	Mean	S.D.
31a	25	1.25	0.84	0.9753	80.0	0.9683	-0.0070	0.9257	0.0313
					90.0	0.9550	-0.0203	0.8764	0.0554
					95.0	0.9418	-0.0335	0.8018	0.1076
31b	25	1.25	0.84	0.9753	80.0	0.9683	-0.0070	0.9247	0.0306
					90.0	0.9550	-0.0203	0.8732	0.0548
					95.0	0.9418	-0.0335	0.7943	0.1076
31c	25	1.25	0.84	0.9753	80.0	0.9683	-0.0070	0.9256	0.0317
					90.0	0.9550	-0.0203	0.8753	0.0560
					95.0	0.9418	-0.0335	0.7984	0.1091

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Confidence Limit Estimate Percentile Values					
				Minimum	10 %	25 %	Median	75 %	90 %
31a	0.97530	311	.80 .90 .95	.83156 .79577 .67687	.88935 .82708 .67687	.90751 .82708 .67687	.91428 .86033 .79841	.96832 .95499 .94184	.96832 .95499 .94184
31b	0.97530	288	.80 .90 .95	.81518 .78157 .67687	.88935 .82708 .67687	.90751 .82708 .67687	.91428 .86033 .79841	.96832 .95499 .94184	.96832 .95499 .94184
31c	0.97530	309	.80 .90 .95	.81299 .77752 .67687	.88935 .82708 .67687	.90751 .82708 .67687	.91428 .86033 .79841	.96832 .95499 .94184	.96832 .95499 .94184

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	RS	CL	\hat{R}^*	D	Mean	S.D.
32a	25	2.50	1.67	0.9753	80.0	0.9562	-0.0191	0.9470	0.0209
					90.0	0.9283	-0.0470	0.9214	0.0195
					95.0	0.9705	-0.0048	0.8841	0.0398
32b	25	2.50	1.67	0.9753	80.0	0.9562	-0.0191	0.9473	0.0211
					90.0	0.9283	-0.0470	0.9218	0.0199
					95.0	0.9705	-0.0048	0.8847	0.0405
32c	25	2.50	1.67	0.9753	80.0	0.9562	-0.0191	0.9472	0.0217
					90.0	0.9395	-0.0358	0.9220	0.0208
					95.0	0.9705	-0.0048	0.8854	0.0408

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Confidence Limit			Estimate			Percentile Values		
				Minimum	10 %	25 %	Median	75 %	90 %	Maximum		
32a	0.97530	79	.80 .90 .95	.88240 .86306 .82272	.92417 .90590 .82272	.94305 .90944 .87577	.95263 .92156 .89354	.95618 .92754 .89572	.95618 .92832 .90735	.98403 .97724 .97049		
32b	0.97530	86	.80 .90 .95	.88265 .86334 .82272	.92417 .90590 .82272	.93511 .90944 .88077	.95263 .92156 .89354	.95618 .92754 .89572	.95618 .92832 .90735	.98403 .97724 .97049		
32c	0.97530	94	.80 .90 .95	.88265 .86334 .82272	.92402 .90572 .82272	.93511 .90944 .87577	.95263 .92156 .89354	.95618 .92754 .89572	.95618 .92832 .90735	.98403 .97724 .97049		

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	RS	CL	R*	D	Mean	S.D.
33a	25	5.00	3.34	0.9753	80.0	0.9711	-0.0042	0.9605	0.0160
					90.0	0.9631	-0.0122	0.9496	0.0140
					95.0	0.9472	-0.0281	0.9366	0.0133
33b	25	5.00	3.34	0.9753	80.0	0.9711	-0.0042	0.9609	0.0158
					90.0	0.9631	-0.0122	0.9498	0.0138
					95.0	0.9472	-0.0281	0.9365	0.0132
33c	25	5.00	3.34	0.9753	80.0	0.9711	-0.0042	0.9604	0.0160
					90.0	0.9631	-0.0122	0.9494	0.0142
					95.0	0.9472	-0.0281	0.9362	0.0128

Table D-1. Simulation Results (concluded).

Case	Rs	Null Runs	CL	Lower Minimum	Confidence Limit 10 %	25 %	Estimate Median	75 %	90 %	Maximum
33a	0.97530	8	.80 .90 .95	.91857 .90748 .89683	.94488 .93475 .91889	.95494 .94449 .93019	.96034 .94990 .93849	.97111 .95768 .94527	.97603 .96309 .94643	.99199 .98855 .98513
33b	0.97530	5	.80 .90 .95	.91860 .90752 .89687	.94488 .93475 .91889	.95494 .94449 .93019	.96138 .95183 .93849	.97111 .95768 .94527	.97603 .96309 .94643	.99199 .98855 .98513
33c	0.97530	3	.80 .90 .95	.91349 .90223 .89149	.94485 .93471 .91889	.95030 .94037 .93019	.96034 .94990 .93849	.97111 .95768 .94527	.97603 .96309 .94643	.99199 .98855 .98513

Table D-2. Additional Simulation Output (concluded)

APPENDIX E

REGRESSION OF OBSERVED ACCURACY

Examination of the tabulated data in Appendix D suggests that a linear relationship is not directly applicable to regression of the accuracy measure, d , on any of the other variables. The terms TT , developed in section IV.B.2, and KK , the number of components, appear to be the most likely candidate independent variables for the model. Plots of these terms versus D suggest that an exponential decreasing or inverse power curve could be used to model procedure accuracy on TT , i.e.

$$D = a \cdot \exp(b \cdot TT) \quad (E.1)$$

$$D = a \cdot TT^b \quad (E.2)$$

where a and b are appropriate constants from the underlying response function. Each of these models can be easily transformed into a linear model by taking the natural logarithm and redefining variables and coefficients where convenient.

The plots indicate, however, that the system configuration has a strong effect. Therefore, curve fitting will be initially confined to similar configurations in the following groups: Cases 01 to 12, Cases 14 to 17, and Cases 22 to 29. Cases 18 to 21 will be considered only with reservations due to the artificial bias introduced by the particular procedure used which ignored failureless runs.

Cases 30 to 33 will not be fitted since the procedure employed caused the sample to be drawn from two quite unlike distributions and the plots demonstrated an apparent discontinuity between the results of the two methods (see Section II.C and Appendix C). Finally, if the group fits are good, a composite fit will be attempted for Cases 01 to 17 and 22 to 29. This fit would not be expected to be good due to the observed inter-group plot variation.

A. EXPONENTIAL CURVE FIT

Consider the linearized exponential model

$$D' = A + b \cdot TT \quad (E.3)$$

where $D' = \ln(-D)$ and $A = \ln(-a)$. From the Method of Least Squares, expressions for A and b can be derived directly.

$$b = \frac{\sum_{i=1}^n TT_i D'_i - \frac{1}{n} \left[\sum_{i=1}^n TT_i \right] \left[\sum_{i=1}^n D'_i \right]}{\sum_{i=1}^n TT_i^2 - \frac{1}{n} \left[\sum_{i=1}^n TT_i \right]^2}$$

$$A = \frac{1}{n} \left[\sum_{i=1}^n D'_i \right] - \frac{b}{n} \left[\sum_{i=1}^n TT_i \right]$$

and the coefficient of determination [Ref. 6] is

$$r^2 = \frac{\left[\sum_{i=1}^n TT_i D'_i - \frac{1}{n} \left[\sum_{i=1}^n D'_i \right] \left[\sum_{i=1}^n TT_i \right] \right]^2}{\left[\sum_{i=1}^n TT_i^2 - \frac{1}{n} \left[\sum_{i=1}^n TT_i \right]^2 \right] \left[\sum_{i=1}^n D'^2_i - \frac{1}{n} \left[\sum_{i=1}^n D'_i \right]^2 \right]}$$

The results of the exponential curve fit were as follows:

<u>Cases</u>	<u>α</u>	<u>a</u>	<u>b</u>	<u>r²</u>
01-12	.20	-0.040	-0.065	0.690
	.10	-0.094	-0.076	0.724
	.05	-0.073	-0.048	0.738
14-17	.20	-0.118	-0.290	0.942
	.10	-0.301	-0.282	0.938
	.05	-0.480	-0.260	0.988
22-29	.20	-0.049	-0.156	0.896
	.10	-0.153	-0.170	0.899
	.05	-0.346	-0.216	0.917

B. INVERSE POWER CURVE FIT

The power curve may be linearized to the form

$$D' = A + b \cdot TT' \quad (E.3)$$

where A and D' are defined as before and $TT' = \ln(TT)$. The coefficients, A and b, and the coefficient of determination may be found as described in the preceding section with the substitution of TT' for TT where occurring.

The results of the power curve fit were as follows:

<u>Cases</u>	<u>α</u>	<u>a</u>	<u>b</u>	<u>r²</u>
01-12	.20	-0.130	-0.868	0.844
	.10	-0.368	-1.016	0.883
	.05	-0.780	-1.124	0.893
14-17	.20	-0.664	-2.085	0.986
	.10	-1.589	-2.016	0.972
	.05	-2.110	-1.828	0.992
22-29	.20	-0.159	-1.305	0.989
	.10	-0.543	-1.412	0.977
	.05	-1.041	-1.430	0.980
01-17	.20	-0.136	-1.060	0.648
22-29	.10	-0.459	-1.228	0.793
	.05	-0.936	-1.303	0.845

Comparing these results with those of the exponential curve fit indicates that the power curve provides a better fit for the observed simulation data. However, the variation in the coefficients between different system

configurations implies that, while the fit may be good within like groups, the fit is much more approximate across the groups. This is borne out by the composite fit listed last in the preceding table.

C. REGRESSION OF D ON TT AND KK

For this regression the model chosen was a combined power curve

$$D = a \cdot TT^b \cdot KK^c \quad (E.5)$$

which linearizes to

$$D' = A + b \cdot TT' + c \cdot KK' \quad (E.6)$$

where D' , A , and TT' are defined as before and $KK' = \ln(KK)$.

Solving for the coefficient values by the Method of Least Squares yields the following:

$$c = \frac{\left[\frac{S1 \cdot S2}{n} - S6 \right] \left[\frac{S1 \cdot S3}{n} - S8 \right] - \left[\frac{S2 \cdot S3}{n} - S7 \right] \left[\frac{S1^2}{n} - S4 \right]}{\left[\frac{S1^2}{n} - S4 \right] \left[\frac{S3^2}{n} - S5 \right] - \left[\frac{S1 \cdot S3}{n} - S8 \right]^2}$$

$$b = \frac{\left[\frac{S1 \cdot S2}{n} - S6 \right] - c \left[\frac{S1 \cdot S3}{n} - S8 \right]}{\frac{S1^2}{n} - S4}$$

$$a = \exp \left[\frac{1}{n} S2 - \frac{b}{n} S1 - \frac{c}{n} S3 \right]$$

where

$$\begin{aligned} S1 &= \sum_{i=1}^n TT'_i & S2 &= \sum_{i=1}^n D'_i & S3 &= \sum_{i=1}^n KK'_i \\ S4 &= \sum_{i=1}^n TT'^2_i & S5 &= \sum_{i=1}^n KK'^2_i & S6 &= \sum_{i=1}^n [TT'_i \cdot D'_i] \end{aligned}$$

$$S7 = \sum_{i=1}^n [D'_i \cdot KK'_i] \quad \text{and} \quad S8 = \sum_{i=1}^n [TT'_i \cdot KK'_i]$$

From the data for cases 01 through 17 and 22 through 29, the following values for a, b, and c were obtained:

$\alpha = 0.20$	$a = -0.080$	$b = -1.028$	$c = 0.129$
$\alpha = 0.10$	$a = -0.386$	$b = -1.200$	$c = 0.033$
$\alpha = 0.05$	$a = -1.038$	$b = -1.279$	$c = -0.043$

An approximate coefficient of determination was calculated by defining

$$TM = TT \cdot KK^{c/b}$$

so that

$$D = a \cdot TM^b$$

which is the inverse power curve model and the two variable procedure of Section B can be used. This yielded an r^2 value of 0.626 for $\alpha = 0.2$, which shows no improvement of fit for the three variable approach.

Appendix D lists TM values for $\alpha = 0.2$ for information.

D. REGRESSION OF STANDARD DEVIATION

Examination of the sample standard deviation values listed in Appendix D also suggests that the power curve form would be appropriate to model the standard deviation on TT.

$$S.D. = a \cdot TT^b$$

Proceeding as in Section B, the following values are obtained:

$\alpha = 0.20$	$a = 0.094$	$b = -0.300$	$r^2 = 0.523$
$\alpha = 0.10$	$a = 0.075$	$b = -0.219$	$r^2 = 0.374$
$\alpha = 0.05$	$a = 0.072$	$b = -0.215$	$r^2 = 0.408$

which indicates a poor fit at best.

However, the data suggests that the sample standard deviation may be better predicted as a function of the number of component tests, i.e.

$$\text{S.D.} = a \bullet N^b$$

where N represents the "equivalent" number of system tests as derived in Section IV.B.2 for use in Equation (4.3).

The regression yields the following values:

$\alpha = 0.20$	$a = 0.213$	$b = -0.391$	$r^2 = 0.853$
$\alpha = 0.10$	$a = 0.186$	$b = -0.365$	$r^2 = 0.825$
$\alpha = 0.05$	$a = 0.164$	$b = -0.339$	$r^2 = 0.799$

This fit is better and implies that the primary mechanism to reduce the sample standard deviation is to increase the number of component tests. As the data base is limited, little more than that may be averred at this level of analysis.

APPENDIX F

COMPUTER PROGRAM

The following pages contain a representative listing of the main computer program utilized in the simulation. Three proprietary routines were used for binomial random variate generation (GGBIN), inverse Chi-Square value determination (MDCHI), and ordering of the estimate arrays (VSORTA). Reference 4 provides information concerning the contents and use of these routines. Equivalent routines could be used without penalty to either accuracy or consistence of the simulation results.

The program was written in the FORTRAN IV language for use on the IBM-360 computer system installed at the Naval Postgraduate School.

Portions of the program must be changed as the subassembly configurations are varied in order to tailor the computations to the specific system being modeled.


```

DIMENSION EREL(1000,5),ZZ(99),KAPN(20),REL(100),FAIL(100),
1 TRIAL(100),RHAT(100),PROB(5),XMEAN(5),XVAR(5),XSD(5)
DATA PROB/0.2,0.1,0.05,0.9,0.9/
READ(5,5) JSD
5 FORMAT(I10)
JSED = JSD
C
C K1 = NUMBER OF SINGLE COMPONENTS IN SERIES
C K2 = NUMBER OF SUBASSEMBLIES IN SERIES
C
10 READ(5,10) K1,K2
   FORMAT(2I5)
   NTOT = K1
   IF (K2.LT.1) GO TO 21
   KAPN(I) = NUMBER OF COMPONENTS IN THE I-TH SUBASSEMBLY
C
C
12 READ(5,12) (KAPN(I), I=1,K2)
   FORMAT(16I5)
   DO 20 I=1,K2
   NTOT = NTOT+KAPN(I)
20 CONTINUE
21 CONTINUE
C
C READ IN COMPONENT RELIABILITIES
C
C K = K1+1
READ(5,30) (REL(I), I=1,K1)
IF (K2.LT.1) GO TO 25
READ(5,30) (REL(I), I=K,NTOT)
25 CONTINUE
C
C READ IN NUMBER OF TRIALS PERFORMED UPON EACH COMPONENT
C
30 FCRMAT(8F10.4)
   KNOT = 0
   T1 = 0.0
   T2 = 0.0
C
C COMPUTE T VALUES FOR SERIES COMPONENTS
C
DO 38 I=1,K1
T1 = T1+TRIAL(I)*(1.0-REL(I))
38 CONTINUE
C
C PERFORM 1000 SIMULATIONS OF COMPONENT TEST DATA AND ESTIMATE SYSTEM REL.
C

```

SREL001
SREL002
SREL003
SREL004
SREL005
SREL006
SREL
SREL007
SREL008
SREL009
SREL010
SREL
SREL011
SREL012
SREL013
SREL014
SREL015
SREL016
SREL
SREL017
SREL018
SREL019
SREL020
SREL021
SREL
SREL022
SREL023
SREL024
SREL025
SREL
SREL025
SREL026
SREL027
SREL
SREL


```

40 DO 120 K=1,1000
   ZK1S = 0.0
   ZK2S = 0.0
   DO 45 N=1,K1
      DETERMINE NUMBER OF FAILURES FOR THIS COMPONENT (BINOMIALLY DISTRIBUTED)
      G = 1.0-REL(N)
      NTEST = TRIAL(N)
      FAIL(N) = GGBIN(JSD,NTEST,Q)
      FRAT = FAIL(N)/TRIAL(N)
      AA = (2.0*TRIAL(N)-3.0)/(2.0*TRIAL(N)-2.0)
      BB = TRIAL(N)/(TRIAL(N)-1.0)
      ZZ(N) = (AA+BB*FRAT/2.0)*FRAT
      ZK1S = ZK1S+ZZ(N)
      ZK2S = ZK2S+ZZ(N)
45  CC CONTINUE
      IF (K2.LT.1) GO TO 1101
      CC COMPUTE SUBASSEMBLY RESULTS
      KSUM = K1+K2
      NUM = K1+1
      KNUM = NUM
      DO 100 N=NUM,KSUM
      KK = N-K1
      JNUM = KNUM + KAPN(KK)-1
      DO 55 NN=KNUM,JNUM
      Q = 1.0-REL(NN)
      NTEST = TRIAL(NN)
      FAIL(NN) = GGBIN(JSD,NTEST,Q)
      RHAT(NN) = 1.0 - FAIL(NN)/TRIAL(NN)
55  CC CONTINUE
      CC COMPUTE SUBASSEMBLY RELIABILITY ESTIMATES
      IF (KK.GT.1) GO TO 56
      ARHAT = 1.0-(1.0-RHAT(KNUM) *RHAT(KNUM+1)*RHAT ( KNUM+2 ))*(1.0-
1  RFRAT(KNUM+3)*RHAT(KNUM+4)*RHAT(KNUM+5))
      GC TO 90
56  IF (KK.GT.2) GO TO 57
      ARHAT = 1.0-(1.0-RHAT(KNUM)*RHAT(KNUM+1))*(1.0-RHAT(KNUM+2))*
1  RHAT(KNUM+3))*RHAT(KNUM+4)*RHAT(KNUM+5))
      GO TO 90
57  IF (KK.GT.3) GO TO 58
      ARHAT = 1.0-(1.0-RHAT(KNUM) ) * (1.0-RHAT(KNUM+1))
      GC TO 90
58  IF (KK.GT.4) GO TO 59

```

```

SREL035
SREL036
SREL037
SREL038
SREL
SREL039
SREL040
SREL041
SREL044
SREL042
SREL043
SREL045
SREL046
SREL047
SREL048
SREL049
SREL
SREL050
SREL051
SREL052
SREL053
SREL054
SREL055
SREL056
SREL057
SREL058
SREL059
SREL060
SREL061
CASE22
CASE22
CASE22
CASE22
CASE22
CASE22
CASE22
CASE22
CASE22
CASE22

```



```

104 CONTINUE
105 GO TO 112
    ARG = ZK2S**2/ZK1S
C
C
C DETERMINE THE DEGREES OF FREEDOM FOR THE CHI-SQUARE VALUE
    DF = AMAX1(1.0,ARG)*2.0
    ACF = DF
    DF = AINT(DF)
    ADF = ADF - DF
    IF (ADF.GT.0.0) DF=DF+1.0
108 DO 110 I=1,3
C
C GET CHI-SQUARE VALUE
    CALL MDCHI(PROB(I),DF,CHISQ,IER)
    EREL(K,I) = EXP(-DF*ZK2S/CHISQ)
    CONTINUE
110 CCNTINUE
111 EREL(K,4) = ZK1S
112 EREL(K,5) = ZK2S
    CONTINUE
C
120 WRITE(6,114)
114 FCORMAT(6,115) (K,(EREL(K,I),I=1,5),K=1,1000)
115 FORMAT(2(4X,I4,1X,5(G10.4,1X)))
    LA = 1000
    DO 130 I=1,3
C
C ORDER THE ESTIMATES
    CALL VSORTA(EREL(1,I),LA)
    CCNTINUE
130
C
C COMPUTE TRUE SYSTEM RELIABILITY
    SREL = 1.0
    DO 135 N=K1,NTOT
    RHAT(N) = REL(N)
    CONTINUE
135 DO 140 N=1,K1
    SREL = SREL*REL(N)
    CONTINUE
140 IF (K2.LT.1) GO TO 147
    KNUM = NUM
    DO 146 N=NUM,KSUM
    KK = N-K1
    IF (KK.GT.1) GO TO 141

```

SCHG C
SCHG D
SCHG

SREL085
SREL086
SREL087
SREL088
SREL089
SREL093

SREL094
SREL095
SREL096
SREL096A
SREL097
SREL098
SREL099
SREL100
SREL101
SREL102
SREL103
SREL104
SREL105

SREL106
SREL107

SREL108
CASE22
CASE22
SREL109
SREL110
SREL111
SREL112
SREL113
SREL114
SREL115
CASE22

CASE22
CASE22
CASE22

```

ARHAT = 1.0-(1.0-RHAT(KNUM) *RHAT(KNUM+1))*RHAT (KNUM+2))*(1.0-
1RHAT(KNUM+3))*RHAT(KNUM+4)*RHAT(KNUM+5))
SREL = SREL*ARHAT
T2 = TRIAL(KI+1)*(1.0-REL(KI+1)) + TRIAL(KI+2)*(1.0-REL(KI+2))
1 T2 + TRIAL(KI+3)*(1.0-REL(KI+3))
T2 = T2/(1.0-REL(KI+1))*REL(KI+2) + REL(KI+3)
T3 = TRIAL(KI+4)*(1.0-REL(KI+4)) + TRIAL(KI+5)*(1.0-REL(KI+5))
1 T3 + TRIAL(KI+6)*(1.0-REL(KI+6))
T3 = T3/(1.0-REL(KI+4))*REL(KI+5) + REL(KI+6)
T2 = Sqrt(T2*T3)*(1.0-ARHAT)
GO TO 145

```

CASE22
CASE22
CASE22
CASE22

```

141 IF (KK.GT.2) GO TO 142
ARHAT = 1.0-(1.0-RHAT(KNUM)*RHAT(KNUM+1))*(1.0-RHAT(KNUM+2))*
1RHAT(KNUM+3))*(1.0-RHAT(KNUM+4))*RHAT(KNUM+5))
SREL = SREL*ARHAT
T3 = TRIAL(KI+7)*(1.0-REL(KI+7)) + TRIAL(KI+8)*(1.0-REL(KI+8))
T3 = T3/(1.0-REL(KI+7))*REL(KI+8)
T4 = TRIAL(KI+9)*(1.0-REL(KI+9)) + TRIAL(KI+10)*(1.0-REL(KI+10))
T4 = T4/(1.0-REL(KI+9))*REL(KI+10)
T5 = TRIAL(KI+11)*(1.0-REL(KI+11))+TRIAL(KI+12)*(1.0-REL(KI+12))
T5 = T5/(1.0-REL(KI+11))*REL(KI+12)
T2 = T2 + EXP(ALOG(T3*T4*T5)/3.0)*(1.0-ARHAT)
GO TO 145

```

CASE22
CASE22
CASE22
CASE22

```

142 IF (KK.GT.3) GO TO 143
ARHAT = 1.0-(1.0-RHAT(KNUM) ) * (1.0-RHAT(KNUM+1))
SREL = SREL*ARHAT
T2 = T2 + Sqrt(TRIAL(KI+13)*TRIAL(KI+14))*(1.0-ARHAT)
GO TO 145

```

CASE22
CASE22
CASE22
CASE22

```

143 IF (KK.GT.4) GO TO 144
ARHAT(KNUM)*RHAT(KNUM+1))*RHAT(KNUM+2))*RHAT(KNUM+3)
A1 = RHAT(KNUM+3))*RHAT(KNUM+4)
A2 = (RHAT(KNUM+4))*RHAT(KNUM+5))*RHAT(KNUM+6))*#2
A3 = RHAT(KNUM)*RHAT(KNUM+1))*RHAT(KNUM+7))*RHAT(KNUM+8)
A4 = RHAT(KNUM+4))*RHAT(KNUM+8)
A5 = RHAT(KNUM+8)
A6 = RHAT(KNUM+9)
A7 = RHAT(KNUM+4)
A8 = RHAT(KNUM+3)*A8*(1.0-(1.0-A6))*(1.0-A5*A7))
A9 = (ARHAT+(1.0-A1)*A4*A5*A3*A8*(1.0-(1.0-A7))*(1.0-A2*A6)))*A9
ARHAT = (1.0-A6)*A7+A6*(1.0-A7)
BRHAT = (1.0-A6)*A2*A3*A4*A5*A8
BRHAT = BRHAT+A6*A7*A8*(1.0-(1.0-A1))*(1.0-A4))*A2*A3*A5
ARHAT = ARHAT+(1.0-A9)*BRHAT
SREL = SREL*ARHAT
T2 = T2 + TRIAL(KI+15)*(1.0-ARHAT)
GO TO 145

```

CASE22
CASE22

```

144 ARHAT = 0.0

```



```

NUM = KNUM-2
DO 1144 I=2,17
ARHAT = ARHAT+RHAT(NNUM+I)*#I
CONTINUE
1144 T2 = T2 + TRIAL(KI+25)*(1.0-ARHAT/16.0)
SREL = SREL*ARHAT/16.0
145 KNUM = KNUM+KAPN(KK)
146 CONTINUE
147 CONTINUE
C
C
C COMPUTE TOTAL TT VALUE FOR SYSTEM
C
TT = T1+T2
DO 149 J=1,3
XMEAN(J) = 0.0
XVAR(J) = 0.0
CCONTINUE
149
C
C
C COMPUTE SAMPLE MEAN AND VARIANCE OF THE ESTIMATES
C
C
DO 160 I=1,1000
DO 150 J=1,3
XMEAN(J) = XMEAN(J)+EREL(I,J)
XVAR(J) = XVAR(J)+EREL(I,J)**2
CONTINUE
150 CONTINUE
160 DO 170 J=1,3
XVAR(J) = (XVAR(J)-XMEAN(J)**2/1000.0)/999.0
XMEAN(J) = XMEAN(J)/1000.0
XSD(J) = SQRT(XVAR(J))
CONTINUE
170 WRITE (6,1006) J,SEED
1006 FORMAT ('1STARTING SEED:',I12//)
194 WRITE (6,194) J,SEED
194 FORMAT ('1FINAL SEED:',I12)
1035 FORMAT ('1COMPNT RELIABILITY TRIALS COMPNT
RELIABILITY TRIALS')
1179 CONTINUE
1036 WRITE (6,1036) (I,REL(I),TRIAL(I), I=1,NTOT)
WRITE (6,1036) (I,REL(I),TRIAL(I), I=1,NTOT)
178 FORMAT ('1H0,20X,'COMPUTATION/SIMULATION RESULTS'///, 'TRUE SYSTEM RES
LIABILITY:',F10.5//)
179 WRITE (6,179) T1,T2,TT
FORMAT ('1T1:',F10.3,5X,'T2:',F10.3,5X,'TT:',F10.3//)
DO 1179 I=1,3
PROB(I) = 1.0-PROB(I)
1179 CONTINUE

```

```

CASE22
CASE22
CASE22
CASE22
SRELI132
SRELI133
SRELI134
SREL
SRELO33
SRELI135
SRELI136
SRELI137
SRELI138
SRELI139
SRELI140
SRELI141
SRELI142
SRELI143
SRELI144
SRELI145
SRELI146
SRELI147
SRELI148
SRELI149
SRELI150
SRELI151
SRELI187
SRELI188
SRELI152
SRELI153
SRELI154
SRELI155
SRELI156
SRELI157
SRELI158
SRELI159
SRELI160
SRELI161
SRELI162
SRELI163
SRELI164

```



```

WRITE (6,180)(PROB(I),I=1,3)
FORMAT (1,CONFIDENCE LEVEL:,16X,1:,3X,1:,3X,3(F8.2,4X)/)
180 ERREL(800,1),EREL(900,2),EREL(950,3)
182 FORMAT (1,CONF. LEVEL PERCENTILE:/,ESTIMATED RELIABILITY',11X,1:'.
1,2X,3F12.5/)
WRITE (6,184) (XMEAN(I), I=1,3)
FORMAT (6,186) (ATTRIBUTES OF, I=1,3)
184 ERREL(800,1),EREL(900,2),EREL(950,3)
186 FORMAT (6,188) (XSD(I), I=1,3)
188 ERREL(800,1),EREL(900,2),EREL(950,3)
190 ERREL(800,1),EREL(900,2),EREL(950,3)
192 ERREL(800,1),EREL(900,2),EREL(950,3)
194 ERREL(800,1),EREL(900,2),EREL(950,3)
196 ERREL(800,1),EREL(900,2),EREL(950,3)
198 ERREL(800,1),EREL(900,2),EREL(950,3)
200 ERREL(800,1),EREL(900,2),EREL(950,3)
END

```

```

EREL1165
SREL1166
SREL1167
SREL1168
SREL1169
SREL1170
SREL1171
SREL1172
SREL1173
SREL1174
SREL1175
SREL1176
SREL1177
SREL1178
SREL1179
SREL1180
SREL1181
SREL1182
SREL1183
SREL1184
SREL1185
SREL1186
SREL1189
SREL1190
SREL1190A
SREL1190B
SREL1190C
SREL1191
SREL1192
SREL1193
SREL1194
SREL1195
SREL1196
SREL1197

```


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